

BEEE203L

CIRCUIT THEORY



Dr. P. Vijayapriya
Professor/ SELECT

MODULE – III

Three Phase Systems

Contents

Network Theorems for AC Circuits

- Review of Balanced System
- Unbalanced System
 - Delta connected loads
 - Three wire star connected loads
 - Four wire star connected loads
- Analysis of unbalanced 3 wire star load
 - Kirchhoff's law
 - Loop current method
 - Star delta conversion method
 - Millman's theorem

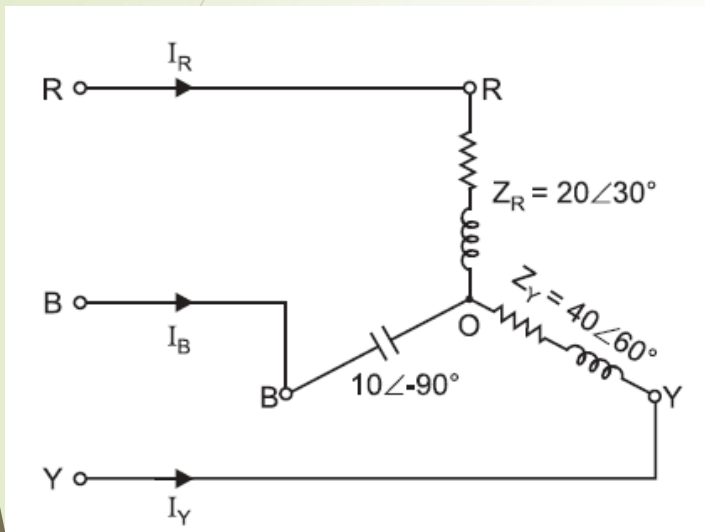
P3.6

A Y-connected load is supplied from a 400 V, 3-phase, 3-wire symmetrical system RYB. The branch circuit impedances are

4

$$Z_R = 10\sqrt{3} + j10 \, \Omega ; Z_Y = 20 + j20\sqrt{3} \, \Omega ; Z_B = 0 - j10 \, \Omega$$

The phase sequence is RYB. Determine the currents in each branch by (i) Kirchhoff's laws (ii) mesh current method



(i) By applying Kirchhoff's laws

The line currents can be obtained as

$$I_R = \frac{V_{RY}Z_B - V_{BR}Z_Y}{Z_RZ_Y + Z_YZ_B + Z_BZ_R}$$

$$Z_RZ_Y + Z_YZ_B + Z_BZ_R = 446.41 + j426.8$$

$$= 617.60/43.7^\circ$$

$$I_R = \frac{400/0^\circ \times 10/-90^\circ - 400/120^\circ \times 40/60^\circ}{617.60/43.7^\circ} = \boxed{26.70/-57.73^\circ \text{ A}}$$

5

$$I_Y = \frac{V_{YB}Z_R - V_{RY}Z_B}{Z_RZ_Y + Z_YZ_B + Z_BZ_R}$$

$$= \frac{400/-120^\circ \times 20/30^\circ - 400/0^\circ \times 10/-90^\circ}{617.60/43.7^\circ} = \boxed{6.47/-133.7^\circ \text{ A}}$$

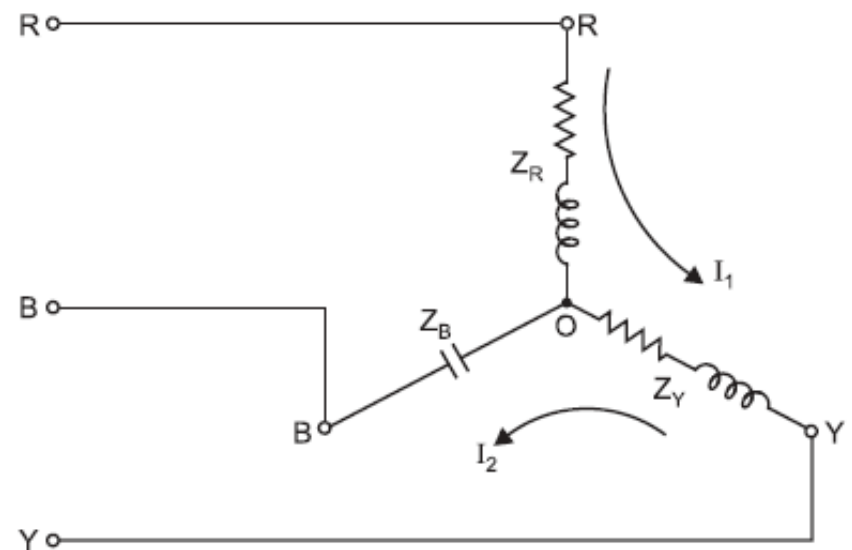
$$I_B = \frac{V_{BR}Z_Y - V_{YB}Z_R}{Z_RZ_Y + Z_YZ_B + Z_BZ_R}$$

$$= \frac{400/120^\circ \times 40/60^\circ - 400/-120^\circ \times 20/30^\circ}{617.60/43.7^\circ} = \boxed{28.96/109.73^\circ \text{ A}}$$

(ii) By mesh current method
Let the mesh currents be I_1 and I_2

$$I_1 = \frac{V_{RY}(Z_Y + Z_B) + V_{YB}Z_Y}{(Z_R + Z_Y)(Z_Y + Z_B) - Z_Y^2}$$

$$I_2 = \frac{V_{YB}(Z_R + Z_Y) + V_{RY}Z_Y}{(Z_R + Z_Y)(Z_Y + Z_B) - Z_Y^2}$$



$$Z_Y + Z_B = 31.74/\underline{50.9^\circ} \Omega$$

$$Z_R + Z_Y = 58.2/\underline{50.1^\circ} \Omega$$

$$\begin{aligned} I_1 &= \frac{400/\underline{0^\circ} \times 31.74/\underline{50.9^\circ} + 400/\underline{-120^\circ} \times 40/\underline{60^\circ}}{58.2/\underline{50.1^\circ} \times 31.74/\underline{50.9^\circ} - 1600/\underline{120^\circ}} \\ &= 14.2 - j22.4 \text{ A} = 26.5/\underline{-57.7^\circ} \text{ A} \end{aligned}$$

$$I_2 = 9.76 - j27.2 \text{ A} = 28.94/\underline{-70.3^\circ} \text{ A}$$

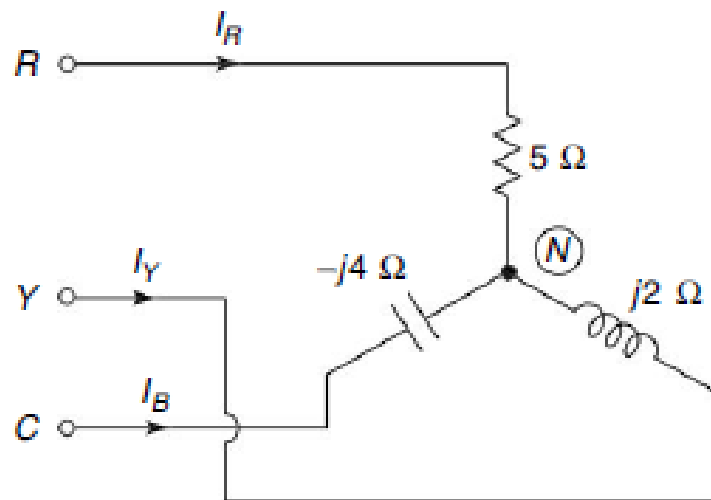
$$I_R = I_1 = 26.5/\underline{-57.7^\circ} \text{ A}$$

$$I_Y = I_2 - I_1 = 6.5/\underline{-133^\circ} \text{ A}$$

$$I_B = -I_2 = 29.8/\underline{109.74^\circ} \text{ A}$$

P3.7**7**

In the circuit of Fig. a symmetrical 3-phase 100 V, three-wire supply feeds an unbalanced star-connected load, with impedances of the load as $Z_R = 5 \angle 0^\circ \Omega$, $Z_Y = 2 \angle 90^\circ \Omega$, $Z_B = 4 \angle -90^\circ \Omega$. Find the line currents and voltage across the impedance using star-delta transformation method.



Solution:

The unbalanced star – connected load can be converted into equivalent delta load by conversion technique

$$Z_{RY} = Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B} = 5 \angle 0^\circ + 2 \angle 90^\circ + \frac{(5 \angle 0^\circ)(2 \angle 90^\circ)}{4 \angle -90^\circ} = 3.2 \angle 38.66^\circ \Omega$$

$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R} = 2 \angle 90^\circ + 4 \angle -90^\circ + \frac{(2 \angle 90^\circ)(4 \angle -90^\circ)}{5 \angle 0^\circ} = 2.56 \angle -51.34^\circ \Omega$$

$$Z_{BR} = Z_B + Z_R + \frac{Z_B Z_R}{Z_Y} = 4 \angle -90^\circ + 5 \angle 0^\circ + \frac{(4 \angle -90^\circ)(5 \angle 0^\circ)}{2 \angle 90^\circ} = 6.4 \angle -141.34^\circ \Omega$$

The equivalent delta load is shown in Fig

$$V_{ph} = V_L = 100 \text{ V}$$

$$V_{RY} = 100 \angle 0^\circ \text{ V}$$

$$V_{YB} = 100 \angle -120^\circ \text{ V}$$

$$V_{BR} = 100 \angle -240^\circ \text{ V}$$

Phase currents

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{100 \angle 0^\circ}{3.2 \angle 38.66^\circ} = 31.25 \angle -38.66^\circ \text{ A}$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{100 \angle -120^\circ}{2.56 \angle -51.34^\circ} = 39.06 \angle -68.66^\circ \text{ A}$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{100 \angle -240^\circ}{6.4 \angle -141.34^\circ} = 15.63 \angle -98.66^\circ \text{ A}$$

$$I_R = I_{RY} - I_{BR} = 31.25 \angle -38.66^\circ - 15.63 \angle -98.66^\circ = 27.06 \angle -8.65^\circ \text{ A}$$

$$I_Y = I_{YB} - I_{RY} = 39.06 \angle -68.66^\circ - 31.25 \angle -38.66^\circ = 19.7 \angle -121.14^\circ \text{ A}$$

$$I_B = I_{BR} - I_{YB} = 15.63 \angle -98.66^\circ - 39.06 \angle -68.66^\circ = 26.69 \angle 128.36^\circ \text{ A}$$

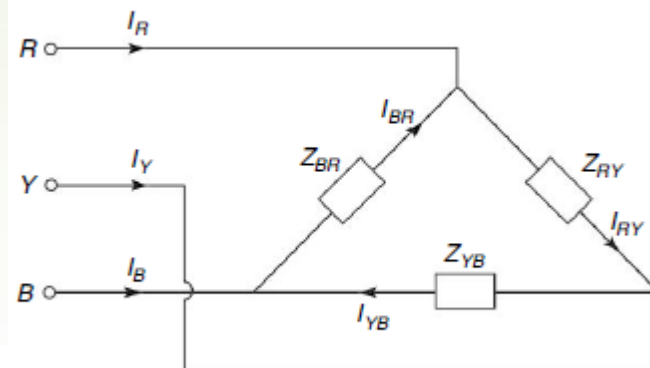


Fig. 8.26

These line currents are equal to the line currents of the original star-connected load. For a star-connected load,

$$I_{ph} = I_L$$

$$I_{RN} = 27.06 \angle -8.65^\circ \text{ A}$$

$$I_{YN} = 19.7 \angle -121.14^\circ \text{ A}$$

$$I_{BN} = 26.69 \angle 128.36^\circ \text{ A}$$

$$V_{RN} = Z_R I_{RN} = (5 \angle 0^\circ)(27.06 \angle -8.65^\circ) = 135.3 \angle -8.65^\circ \text{ V}$$

$$V_{YN} = Z_Y I_{YN} = (2 \angle 90^\circ)(19.7 \angle -121.14^\circ) = 39.4 \angle -31.14^\circ \text{ V}$$

$$V_{BN} = Z_B I_{BN} = (4 \angle -90^\circ)(26.69 \angle 128.36^\circ) = 106.76 \angle 38.36^\circ \text{ V}$$

P3.8 A symmetrical 440 V, 3-phase system supplied a star-connected load with the branch impedances $Z_R = 10 \Omega$ $Z_Y = j5 \Omega$ $Z_B = -j5 \Omega$. Calculate line currents and voltage across each phase impedance by Millman's theorem. The phase sequence is RYB.

10

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

Let O be the neutral point of the supply system.

Let $V_{RO} = 254.03 \angle 0^\circ \text{ V}$

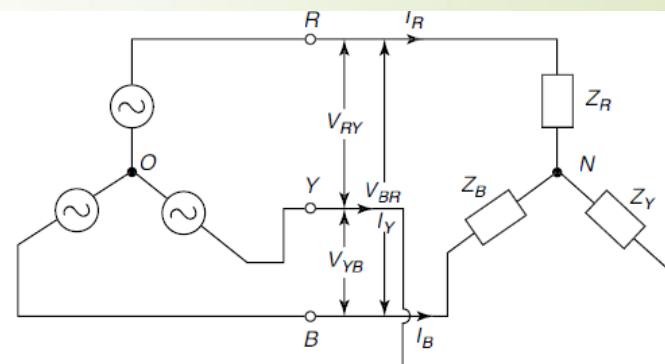
$$V_{YO} = 254.03 \angle -120^\circ \text{ V}$$

$$V_{BO} = 254.03 \angle -240^\circ \text{ V}$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{10} = \frac{1}{10 \angle 0^\circ} = 0.1 \angle 0^\circ \text{ S}$$

$$Y_Y = \frac{1}{Z_Y} = \frac{1}{j5} = \frac{1}{5 \angle 90^\circ} = 0.2 \angle -90^\circ \text{ S}$$

$$Y_B = \frac{1}{Z_B} = \frac{1}{-j5} = \frac{1}{5 \angle -90^\circ} = 0.2 \angle 90^\circ \text{ S}$$



By Millman's theorem,

$$\mathbf{V}_{NO} = \frac{\mathbf{V}_{RO} \mathbf{Y}_R + \mathbf{V}_{YO} \mathbf{Y}_Y + \mathbf{V}_{BO} \mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

$$= \frac{(254.03 \angle 0^\circ)(0.1 \angle 0^\circ) + (254.03 \angle -120^\circ)(0.2 \angle -90^\circ) + (254.03 \angle -240^\circ)(0.2 \angle 90^\circ)}{0.1 \angle 0^\circ + 0.2 \angle -90^\circ + 0.2 \angle 90^\circ}$$

$$= 625.96 \angle 180^\circ \text{ V}$$

Voltage across phase impedance

$$\mathbf{V}_{RN} = \mathbf{V}_{RO} - \mathbf{V}_{NO} = 254.03 \angle 0^\circ - 625.96 \angle 180^\circ = 880 \angle 0^\circ \text{ V}$$

$$\mathbf{V}_{YN} = \mathbf{V}_{YO} - \mathbf{V}_{NO} = 254.03 \angle -120^\circ - 625.96 \angle 180^\circ = 545.29 \angle -23.79^\circ \text{ V}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{BO} - \mathbf{V}_{NO} = 254.03 \angle -240^\circ - 625.96 \angle 180^\circ = 545.29 \angle 23.79^\circ \text{ V}$$

Phase currents / Line currents

$$\mathbf{I}_R = \frac{\mathbf{V}_{RN}}{\mathbf{Z}_R} = \frac{880 \angle 0^\circ}{10 \angle 0^\circ} = 88 \angle 0^\circ \text{ A}$$

$$\mathbf{I}_Y = \frac{\mathbf{V}_{YN}}{\mathbf{Z}_Y} = \frac{545.29 \angle -23.79^\circ}{5 \angle 90^\circ} = 109.06 \angle -113.79^\circ \text{ A}$$

$$\mathbf{I}_B = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B} = \frac{545.29 \angle 23.79^\circ}{5 \angle -90^\circ} = 109.06 \angle 113.79^\circ \text{ A}$$