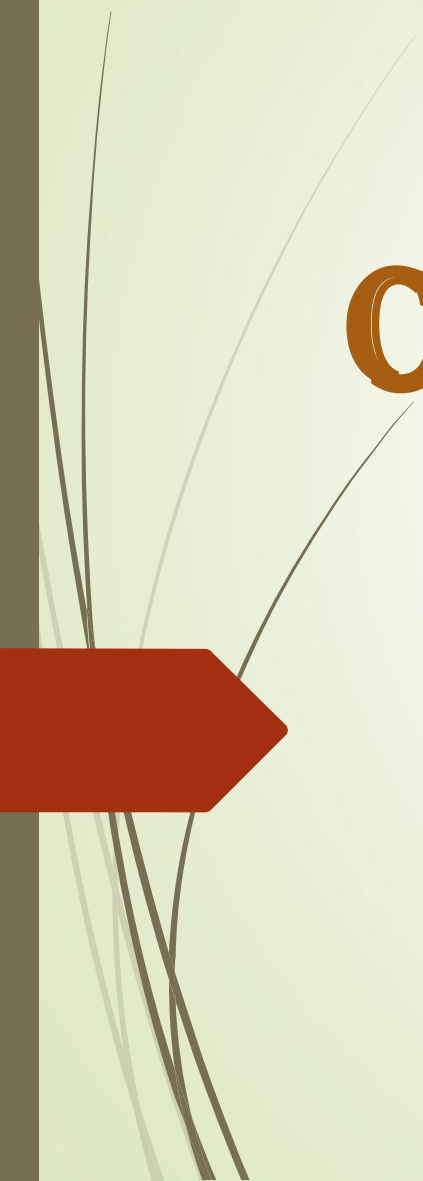




BEEE203L

CIRCUIT THEORY

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MODULE – VII

TWO PORT NETWORKS

Contents

- Open circuit impedance parameters
- Short circuit admittance parameters
- Transmission parameters
- Hybrid parameters
- Relationship between parameter sets
- Interconnection of two port networks

Circuit with dependent sources

Find the Y parameters of the network shown below.

$$\frac{V_1 - V_o}{8} = 2I_1 + \frac{V_o}{2} + \frac{V_o - 0}{4}$$

$$I_1 = \frac{V_1 - V_o}{8}; \text{ therefore,}$$

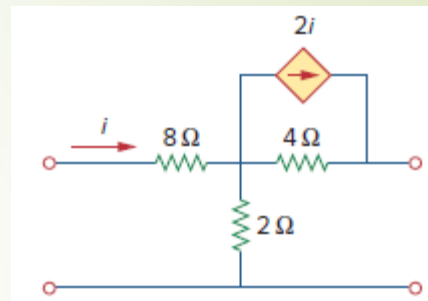
$$0 = \frac{V_1 - V_o}{8} + \frac{3V_o}{4}$$

$$0 = V_1 - V_o + 6V_o \quad \Rightarrow \quad V_1 = -5V_o$$

$$I_1 = \frac{-5V_o - V_o}{8} = -0.75V_o$$

$$y_{11} = \frac{I_1}{V_1} = \frac{-0.75V_o}{-5V_o} = 0.15 \text{ S}$$

$$\frac{V_o - 0}{4} + 2I_1 + I_2 = 0$$



$$-I_2 = 0.25V_o - 1.5V_o = -1.25V_o$$

$$y_{21} = \frac{I_2}{V_1} = \frac{1.25V_o}{-5V_o} = -0.25 \text{ S}$$

$$\frac{0 - V_o}{8} = 2I_1 + \frac{V_o}{2} + \frac{V_o - V_2}{4}$$

But $I_1 = \frac{0 - V_o}{8}$; therefore,

$$0 = -\frac{V_o}{8} + \frac{V_o}{2} + \frac{V_o - V_2}{4}$$

or

$$0 = -V_o + 4V_o + 2V_o - 2V_2 \quad \Rightarrow \quad V_2 = 2.5V_o$$

Hence,

$$y_{12} = \frac{I_1}{V_2} = \frac{-V_o/8}{2.5V_o} = -0.05 \text{ S}$$

At node 2,

$$\frac{V_o - V_2}{4} + 2I_1 + I_2 = 0$$

or

$$-I_2 = 0.25V_o - \frac{1}{4}(2.5V_o) - \frac{2V_o}{8} = -0.625V_o$$

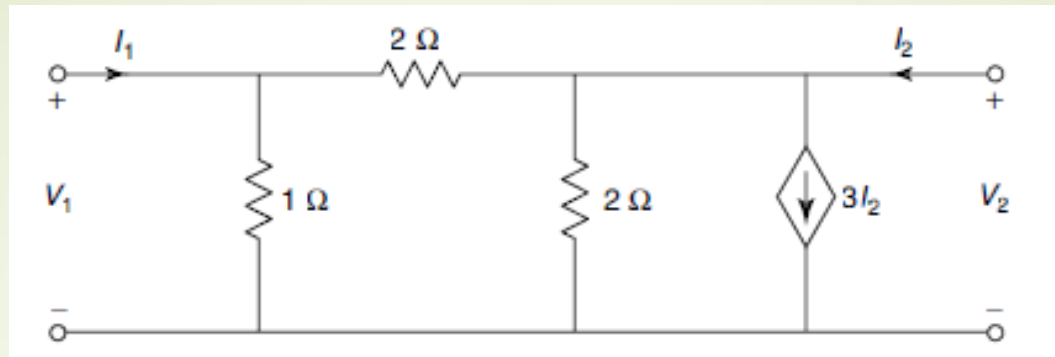
Thus,

$$y_{22} = \frac{I_2}{V_2} = \frac{0.625V_o}{2.5V_o} = 0.25 \text{ S}$$

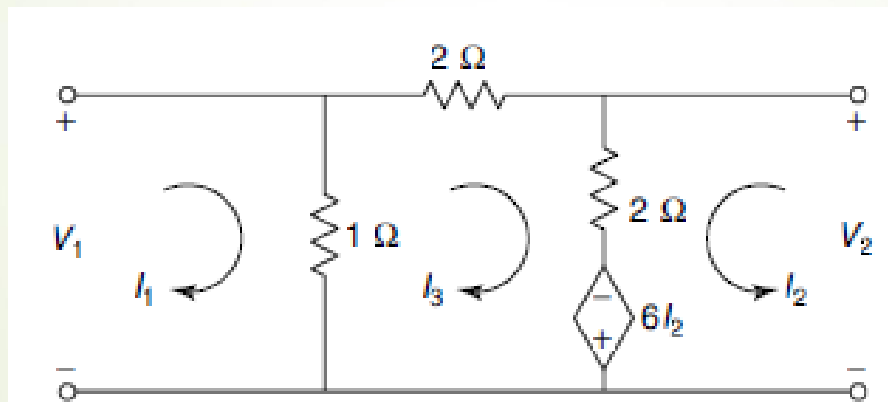
Notice that $y_{12} \neq y_{21}$ in this case, given that the network is not reciprocal.

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For the network shown below, Find Z and Y parameters



The network is redrawn using source transformation technique as



Applying KVL to Mesh 1,

$$V_1 = I_1 - I_3 \quad \dots(i)$$

Applying KVL to Mesh 2,

$$\begin{aligned} V_2 &= 2(I_2 + I_3) - 6I_2 \\ &= -4I_2 + 2I_3 \quad \dots(ii) \end{aligned}$$

Applying KVL to Mesh 3,

$$-(I_3 - I_1) - 2I_3 - 2(I_2 + I_3) + 6I_2 = 0$$

$$5I_3 = I_1 + 4I_2$$

$$I_3 = \frac{1}{5}I_1 + \frac{4}{5}I_2 \quad \dots(\text{iii})$$

Substituting Eq. (iii) in Eq. (i),

$$\begin{aligned} V_1 &= I_1 - \frac{1}{5}I_1 - \frac{4}{5}I_2 \\ &= \frac{4}{5}I_1 - \frac{4}{5}I_2 \end{aligned}$$

Substituting Eq. (iii) in Eq. (ii),

$$\begin{aligned} V_2 &= -4I_2 + 2\left(\frac{1}{5}I_1 + \frac{4}{5}I_2\right) \\ &= \frac{2}{5}I_1 - \frac{12}{5}I_2 \end{aligned}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & -\frac{4}{5} \\ \frac{2}{5} & -\frac{12}{5} \end{bmatrix}$$

Y-parameters

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21} = \left(\frac{4}{5}\right)\left(-\frac{12}{5}\right) - \left(-\frac{4}{5}\right)\left(\frac{2}{5}\right) = -\frac{40}{25} = -\frac{8}{5}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{-\frac{12}{5}}{-\frac{8}{5}} = \frac{3}{2} \text{ } \Omega^{-1},$$

$$Y_{12} = -\frac{Z_{12}}{\Delta Z} = -\frac{-\frac{4}{5}}{-\frac{8}{5}} = -\frac{1}{2} \text{ } \Omega^{-1}$$

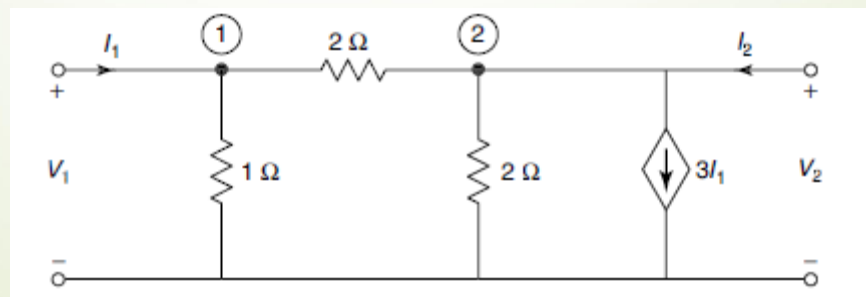
$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -\frac{\frac{2}{5}}{-\frac{8}{5}} = \frac{1}{4} \text{ } \Omega^{-1},$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{\frac{4}{5}}{-\frac{8}{5}} = -\frac{1}{2} \text{ } \Omega^{-1}$$

Hence, the Y-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$$

Determine Y and Z parameters for the network shown



Solution Applying KCL at Node 1,

$$\begin{aligned} I_1 &= \frac{V_1}{1} + \frac{V_1 - V_2}{2} \\ &= 1.5V_1 - 0.5V_2 \end{aligned} \quad \dots(i)$$

Applying KCL at Node 2,

$$\begin{aligned} I_2 &= \frac{V_2}{2} + 3I_1 + \frac{V_2 - V_1}{2} \\ &= \frac{V_2}{2} + 3(1.5V_1 - 0.5V_2) + \frac{V_2 - V_1}{2} \\ &= 0.5V_2 + 4.5V_1 - 1.5V_2 + 0.5V_2 - 0.5V_1 \\ &= 4V_1 - 0.5V_2 \end{aligned} \quad \dots(ii)$$

Comparing Eqs (i) and (ii) with the Y -parameter equation,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix}$$

Z-parameters

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = (1.5)(-0.5) - (-0.5)(4) = 1.25$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = -\frac{0.5}{1.25} = -0.4 \, \Omega,$$

$$Z_{12} = -\frac{Y_{12}}{\Delta Y} = \frac{0.5}{1.25} = 0.4 \, \Omega$$

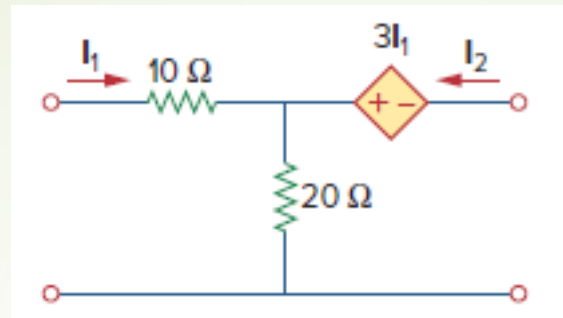
$$Z_{21} = -\frac{Y_{21}}{\Delta Y} = -\frac{4}{1.25} = -3.2 \, \Omega,$$

$$Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{1.5}{1.25} = 1.2 \, \Omega$$

Hence, the Z -parameters are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$$

Find the transmission line parameters of the two port network shown below



Solution:

To determine **A** and **C**, we leave the output port open as in Fig. 19.33(a) so that $I_2 = 0$ and place a voltage source V_1 at the input port. We have

$$V_1 = (10 + 20)I_1 = 30I_1 \quad \text{and} \quad V_2 = 20I_1 - 3I_1 = 17I_1$$

Thus,

$$A = \frac{V_1}{V_2} = \frac{30I_1}{17I_1} = 1.765, \quad C = \frac{I_1}{V_2} = \frac{I_1}{17I_1} = 0.0588 \text{ S}$$

To obtain **B** and **D**, we short-circuit the output port so that $V_2 = 0$ as shown in Fig. 19.33(b) and place a voltage source V_1 at the input port. At node a in the circuit of Fig. 19.33(b), KCL gives

$$\frac{V_1 - V_a}{10} - \frac{V_a}{20} + I_2 = 0 \quad (19.8.1)$$

But $V_a = 3I_1$ and $I_1 = (V_1 - V_a)/10$. Combining these gives

$$V_a = 3I_1 \quad V_1 = 13I_1 \quad (19.8.2)$$

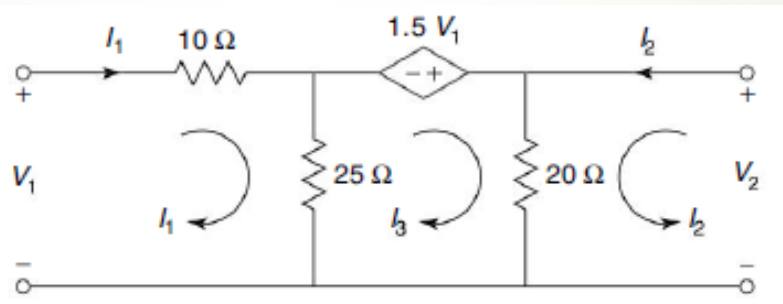
Substituting $V_a = 3I_1$ into Eq. (19.8.1) and replacing the first term with I_1 ,

$$I_1 - \frac{3I_1}{20} + I_2 = 0 \quad \Rightarrow \quad \frac{17}{20} I_1 = -I_2$$

Therefore,

$$\mathbf{D} = -\frac{I_1}{I_2} = \frac{20}{17} = 1.176, \quad \mathbf{B} = \frac{V_1}{I_2} = \frac{-13I_1}{(-17/20)I_1} = 15.29 \, \Omega$$

Find the transmission line parameters of the two port network shown below



Solution Applying KVL to Mesh 1,

$$\begin{aligned} V_1 &= 10I_1 + 25(I_1 - I_3) \\ &= 35I_1 - 25I_3 \end{aligned}$$

Applying KVL to Mesh 2,

$$\begin{aligned} V_2 &= 20(I_2 + I_3) \\ &= 20I_2 + 20I_3 \end{aligned}$$

Applying KVL to Mesh 3,

$$-25(I_3 - I_1) + 1.5 V_1 - 20(I_2 + I_3) = 0$$

$$-25I_3 + 25I_1 + 1.5(35I_1 - 25I_3) - 20I_2 - 20I_3 = 0$$

$$-25I_3 + 25I_1 + 52.5I_1 - 37.5I_3 - 20I_2 - 20I_3 = 0$$

$$82.5I_3 = 77.5I_1 - 20I_2$$

$$I_3 = 0.94I_1 - 0.24I_2$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 35I_1 - 25(0.94I_1 - 0.24I_2)$$

$$= 11.5I_1 + 6I_2$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 20I_2 + 20(0.94I_1 - 0.24I_2)$$

$$= 18.8I_1 + 15.2I_2$$

From Eq. (v),

$$I_1 = 0.053 V_2 - 0.81I_2$$

Substituting Eq. (vi) in Eq. (iv),

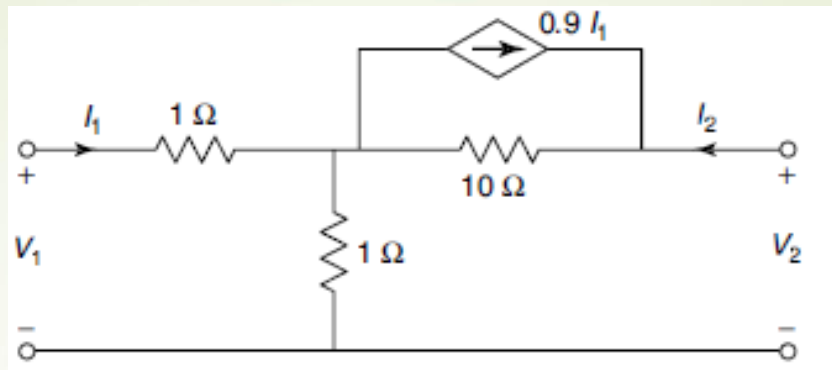
$$V_1 = 11.5(0.053 V_2 - 0.81I_2) + 6I_2$$

$$= 0.61 V_2 - 3.32I_2$$

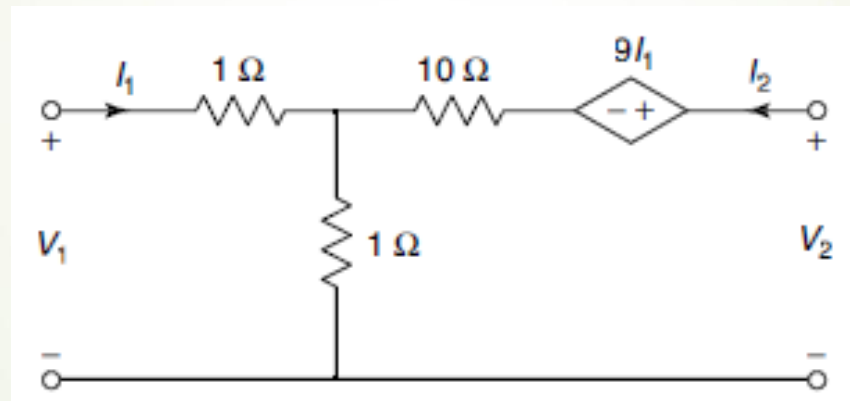
Comparing Eqs (vi) and (vii) with $ABCD$ parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.61 & -3.32 \\ 0.053 & -0.81 \end{bmatrix}$$

Find the Z parameters for the network shown below



Applying source transformation



Applying KVL to Mesh 1,

$$V_1 = 2I_1 + I_2 \quad \dots(i)$$

Applying KVL to Mesh 2,

$$\begin{aligned} V_2 &= 9I_1 + 10I_2 + 1(I_1 + I_2) \\ &= 10I_1 + 11I_2 \quad \dots(ii) \end{aligned}$$

Comparing Eqs (i) and (ii) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 10 & 11 \end{bmatrix}$$

Y-parameters

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21} = (2)(11) - (1)(10) = 22 - 10 = 12$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{11}{12} \text{ } \Omega,$$

$$Y_{12} = -\frac{Z_{12}}{\Delta Z} = -\frac{1}{12} \text{ } \Omega$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -\frac{10}{12} = -\frac{5}{6} \text{ } \Omega,$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{2}{12} = \frac{1}{6} \text{ } \Omega$$

Hence, the Y-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{12} & -\frac{1}{12} \\ -\frac{5}{6} & \frac{1}{6} \end{bmatrix}$$

h-parameters

$$h_{11} = \frac{\Delta Z}{Z_{22}} = \frac{12}{11} \text{ } \Omega,$$

$$h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{1}{11}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} = -\frac{10}{11},$$

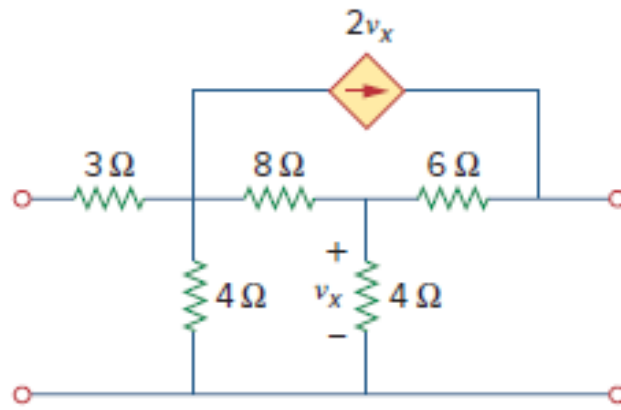
$$h_{22} = \frac{1}{Z_{22}} = \frac{1}{11} \text{ } \Omega$$

Hence, h-parameters are

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{12}{11} & \frac{1}{11} \\ -\frac{10}{11} & \frac{1}{11} \end{bmatrix}$$

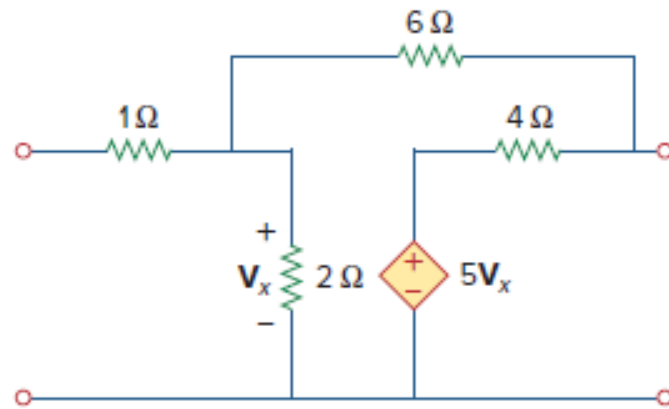
Obtain the h parameters for the network shown below

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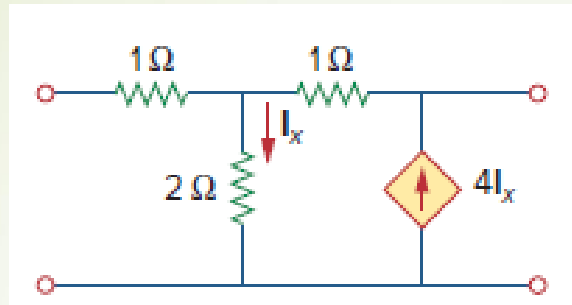
Answer: $h_{11} = 4.238 \Omega$, $h_{21} = -0.6190$, $h_{12} = -0.7143$, $h_{22} = -0.1429 \text{ S}$.

Obtain the ABCD parameters for the network shown

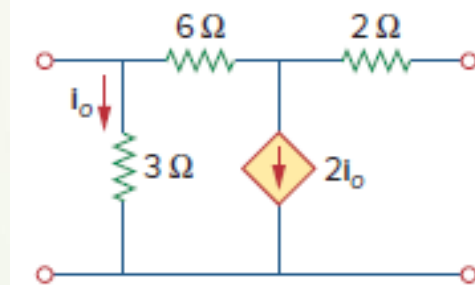


Answer: $\begin{bmatrix} 0.3235 & 1.176 \Omega \\ 0.02941 \text{ S} & 0.4706 \end{bmatrix}$

Find the transmission parameters for the circuit given below



Obtain the Y parameters for the circuit shown below



Answer: $y_{11} = 625 \text{ mS}$, $y_{12} = -125 \text{ mS}$, $y_{21} = 375 \text{ mS}$, $y_{22} = 125 \text{ mS}$.