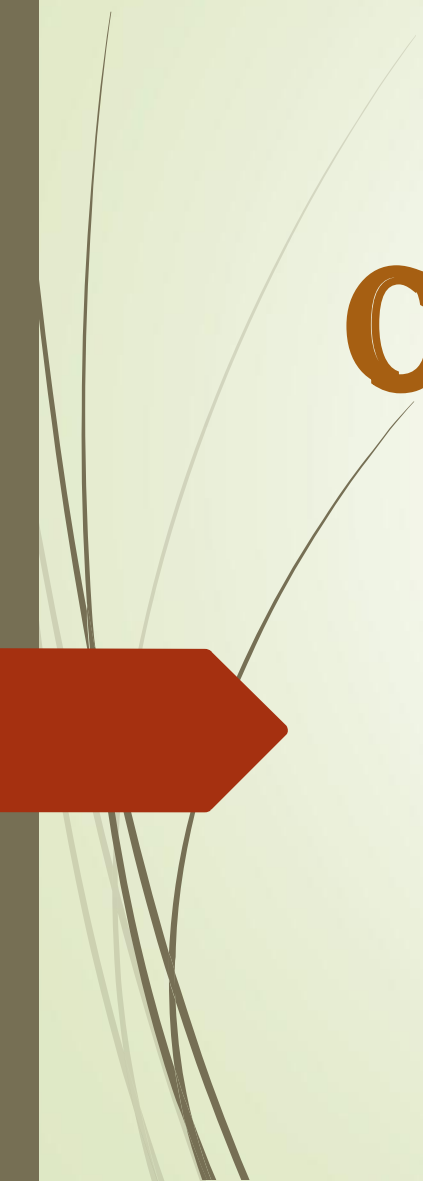




BEEE203L

CIRCUIT THEORY

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MODULE – VII

TWO PORT NETWORKS

Contents

- Open circuit impedance parameters
- Short circuit admittance parameters
- Transmission parameters
- Hybrid parameters
- Relationship between parameter sets
- Interconnection of two port networks

SHORT-CIRCUIT ADMITTANCE PARAMETER. (Y PARAMETERS)

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For some two-port networks, the z-parameters do not exist ex: ideal transformer

The other set of parameters can be obtained by expressing the terminal currents in terms of the terminal voltages known as *admittance parameters* or *y-parameters*

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

The values of the parameters can be determined by setting $V_1 = 0$ (input port short-circuited) or $V_2 = 0$ (output port short-circuited)

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}, \quad y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

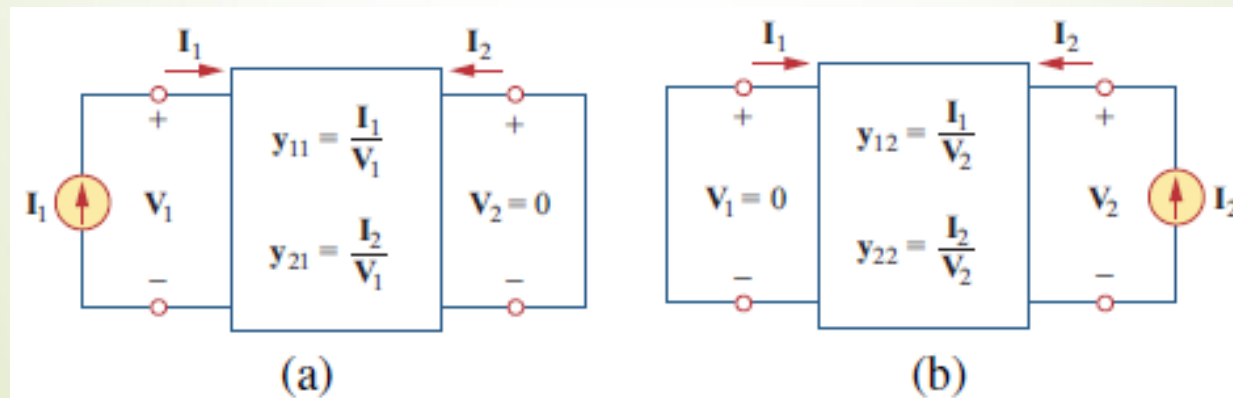
$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}, \quad y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

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Since the Y parameters are obtained by short-circuiting the input or output port, they are also called the short-circuit admittance parameters

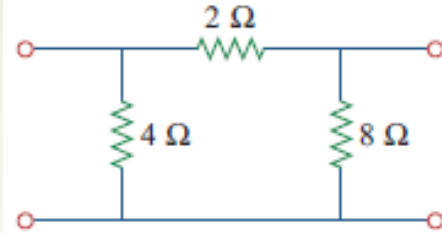
- Y_{11} is Short-circuit input admittance
- Y_{12} is Short-circuit transfer admittance from port 2 to port 1
- Y_{21} is Short-circuit transfer admittance from port 1 to port 2
- Y_{22} is Short-circuit output admittance
- Y_{11} and Y_{22} are called *driving-point admittances*
- Y_{12} and Y_{21} are called *transfer admittances*



For a two-port network that is linear and has no dependent sources, the transfer admittances are equal ($Y_{12} = Y_{21}$)

P.7.6 Obtain the y parameters for the network shown below

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Solution: To find Y_{11} and Y_{21} , short-circuit the output port and connect a current source I_1 to the input port

$$V_1 = (2 \parallel 4)I_1 = \frac{4}{3}I_1,$$

So

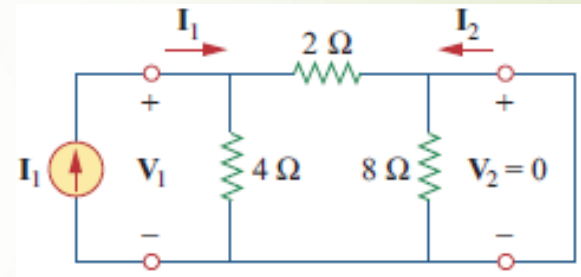
$$y_{11} = \frac{I_1}{V_1} = \frac{I_1}{4/3I_1} = 0.75 \text{ S}$$

from the current divide rule

$$I_2 = -\frac{4}{4+2}I_1 = -\frac{2}{3}I_1,$$

So

$$y_{21} = \frac{I_2}{V_1} = \frac{-2/3I_1}{4/3I_1} = -0.5 \text{ S}$$



$$y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

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To find Y_{12} and Y_{22} , short-circuit the input port and connect a current source I_2 to the input port

So

$$V_2 = (8 \parallel 2)I_2 = \frac{8}{5}I_2,$$

$$y_{22} = \frac{I_2}{V_2} = \frac{I_2}{8/5I_2} = 0.625 \text{ S}$$

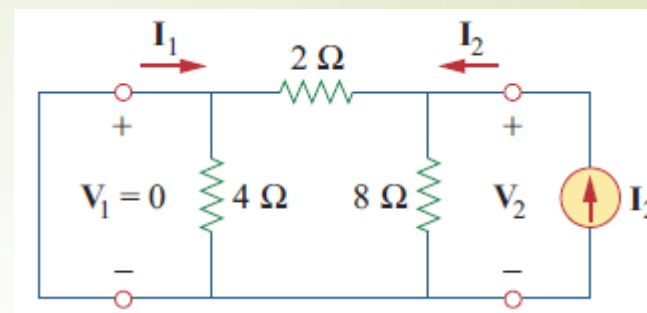
from the current divide rule

$$I_1 = -\frac{8}{8+2}I_2 = -\frac{4}{5}I_2,$$

So

$$y_{12} = \frac{I_1}{V_2} = \frac{-4/5I_2}{8/5I_2} = -0.5 \text{ S}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

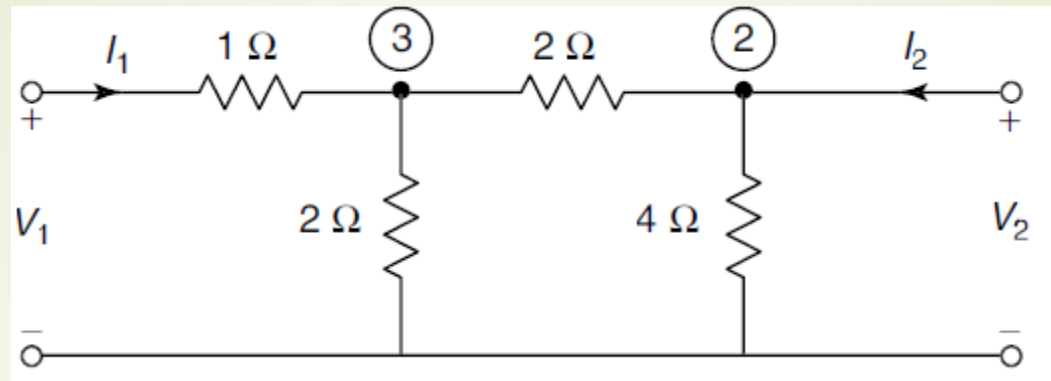


$$y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

$$y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

P.7.7

Determine Y-parameters for the network shown in Fig. Determine whether the network is symmetrical and reciprocal.

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Solution: - Applying KCL at Node 3

$$I_1 = \frac{V_1 - V_3}{1} = V_1 - V_3$$

$$I_1 = \frac{V_3}{2} + \frac{V_3 - V_2}{2} = V_3 - \frac{V_2}{2}$$

or

$$V_1 - V_3 = V_3 - \frac{V_2}{2} \implies V_3 = \frac{V_1}{2} + \frac{V_2}{4}$$

So

$$I_1 = \frac{V_1}{2} + \frac{V_2}{4} - \frac{V_2}{2} = \frac{V_1}{2} - \frac{V_2}{4}$$

Applying KCL at node 2

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$$I_2 = \frac{V_2}{4} + \frac{V_2 - V_3}{2}$$

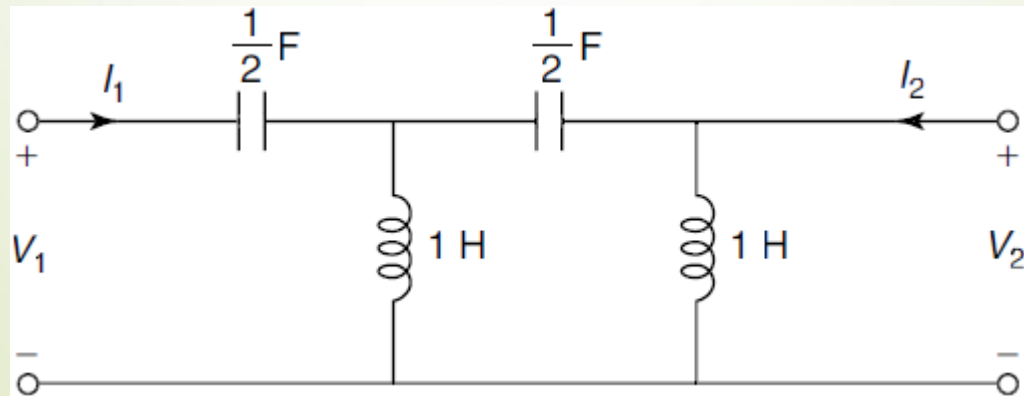
$$= \frac{3}{4}V_2 - \frac{V_3}{2}$$

Or
$$I_2 = \frac{3}{4}V_2 - \frac{1}{2}\left(\frac{V_1}{2} + \frac{V_2}{4}\right) = -\frac{V_1}{4} + \frac{5V_2}{8}$$

Writing in matrix form

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{5}{8} \end{bmatrix}$$

P.7.8 Determine the Y parameters of the network shown in Figure



The transformed circuit is shown and hence

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$$I_1 = \frac{V_1 - V_3}{\frac{2}{s}}$$

$$= \frac{s}{2} V_1 - \frac{s}{2} V_3$$

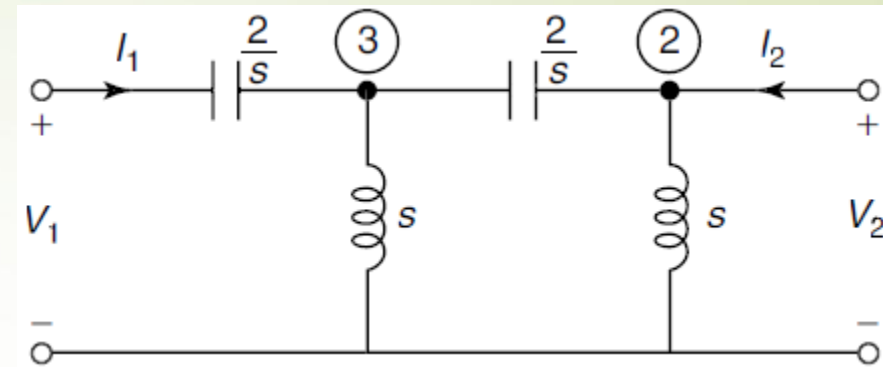
Applying KCL at node 3

$$\frac{s}{2}(V_1 - V_3) = \frac{V_3}{s} + \frac{s}{2}(V_3 - V_2)$$

$$\frac{s}{2} V_3 + \frac{1}{s} V_3 + \frac{s}{2} V_3 = \frac{s}{2} V_1 + \frac{s}{2} V_2$$

$$V_3 = \frac{s^2}{2(s^2 + 1)} V_1 + \frac{s^2}{2(s^2 + 1)} V_2$$

$$I_1 = \frac{s}{2} V_1 - \frac{s}{2} \left[\frac{s^2}{2(s^2 + 1)} V_1 + \frac{s^2}{2(s^2 + 1)} V_2 \right] = \left[\frac{s}{2} - \frac{s^3}{4(s^2 + 1)} \right] V_1 - \frac{s^3}{4(s^2 + 1)} V_2$$



$$I_1 := \frac{s^3 + 2s}{4(s^2 + 1)} V_1 - \frac{s^3}{4(s^2 + 1)} V_2$$

Applying KCL at node 2

$$I_2 = \frac{V_2}{s} + \frac{s}{2}(V_2 - V_3) = \frac{s^2 + 2}{2s} V_2 - \frac{s}{2} V_3$$

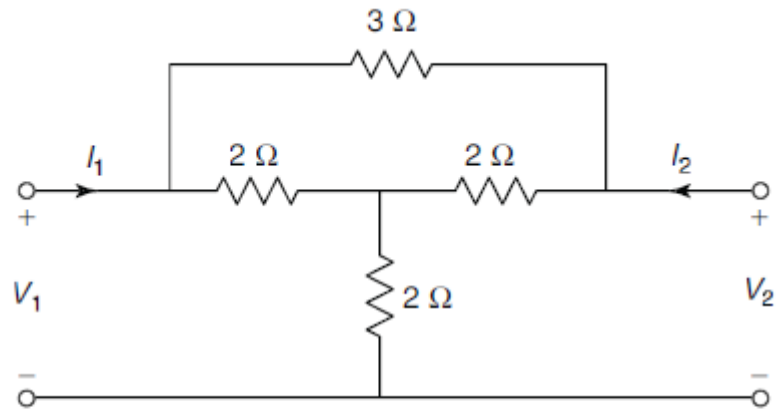
Substituting for V_3 we get

$$\begin{aligned} I_2 &= \frac{s^2 + 2}{2s} V_2 - \frac{s}{2} \left[\frac{s^2}{2(s^2 + 1)} V_1 + \frac{s^2}{2(s^2 + 1)} V_2 \right] \\ &= -\frac{s^3}{4(s^2 + 1)} V_1 + \left[\frac{s^2 + 2}{2s} - \frac{s^3}{4(s^2 + 1)} \right] V_2 \\ &= -\frac{s^3}{4(s^2 + 1)} V_1 + \frac{s^4 + 6s^2 + 4}{4s(s^2 + 1)} V_2 \end{aligned}$$

So

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{s^3 + 2s}{4(s^2 + 1)} & -\frac{s^3}{4(s^2 + 1)} \\ -\frac{s^3}{4(s^2 + 1)} & \frac{s^4 + 6s^2 + 4}{4s(s^2 + 1)} \end{bmatrix}$$

P.7.9 Obtain the Y parameters for the network shown below



$$\text{Ans: } y_{11} = \frac{2}{3} \text{ U}, y_{12} = -\frac{1}{2} \text{ U}, y_{21} = -\frac{1}{2} \text{ U}, y_{22} = \frac{2}{3} \text{ U}$$

Applications

- The impedance and admittance parameters are collectively referred to as *immittance* parameters
- Impedance and admittance parameters are commonly used in the synthesis of filters.
- They are also useful in the design and analysis of impedance-matching networks and power distribution networks.