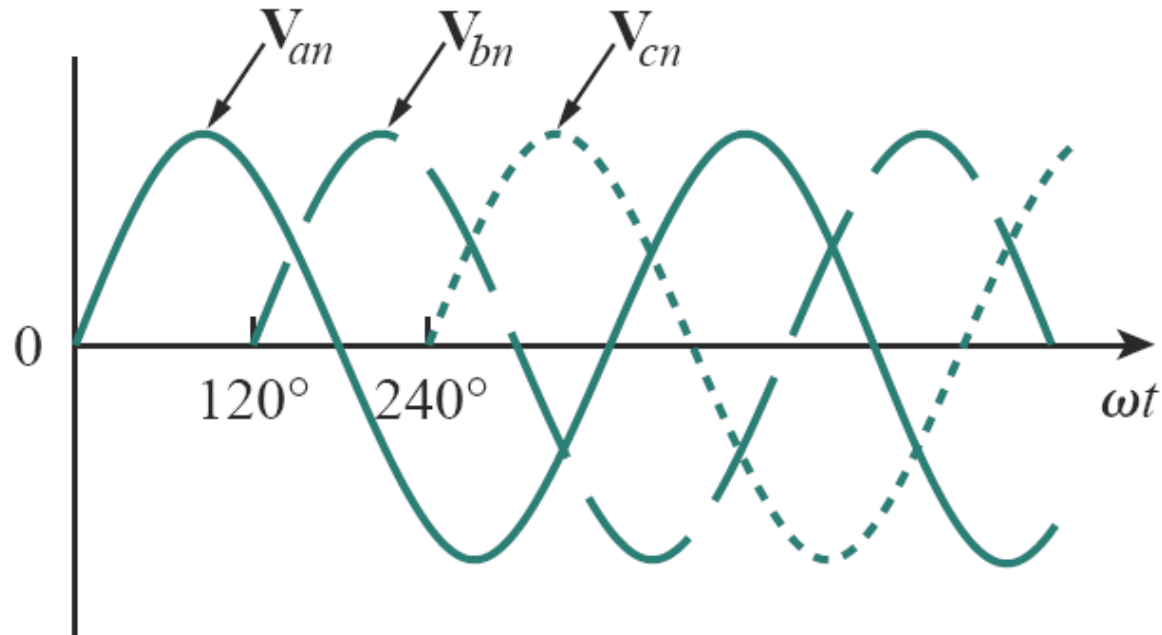
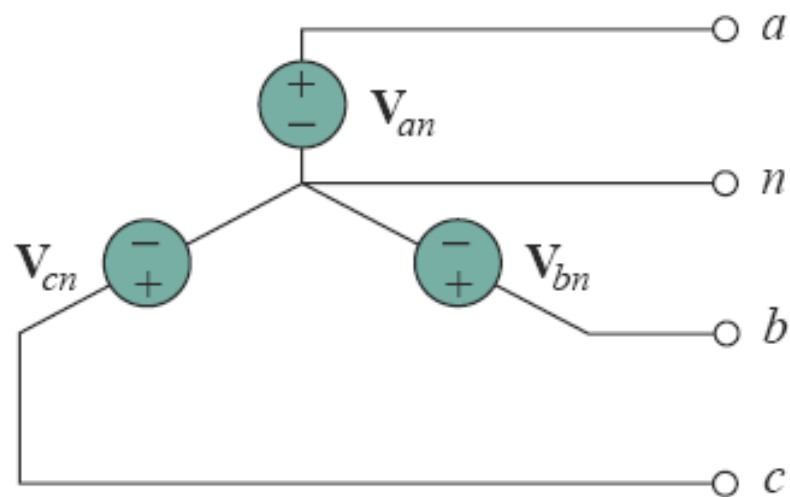


Three-phase systems

Advantages of three-phase systems over single-phase supplies include:

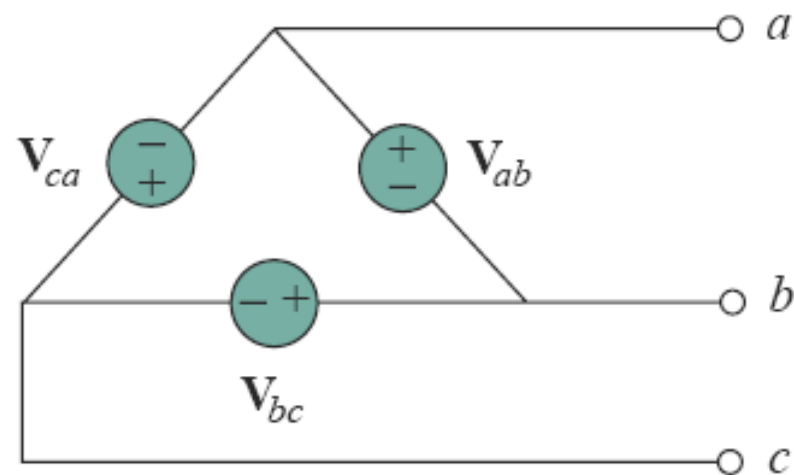
- (i) For a given amount of power transmitted through a system, the three-phase system requires conductors with a smaller cross sectional area. This means a saving of copper (or aluminium) and thus the original installation costs are less.
- (ii) The instantaneous power in a three-phase system can be constant (not pulsating).
- (iii) It is also used to drive large motors and other heavy loads.



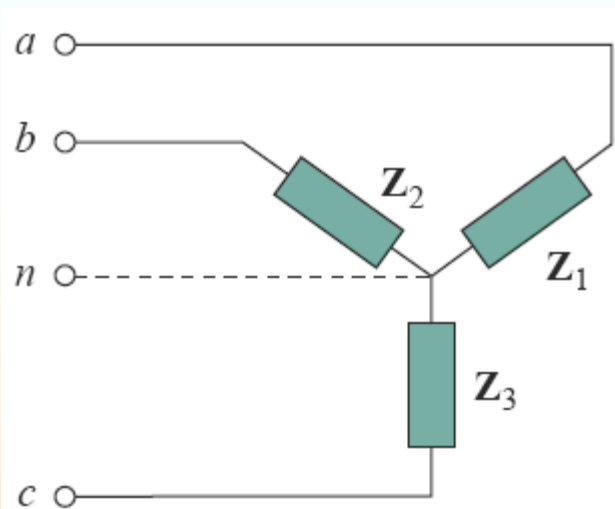


(a)

Three phase source

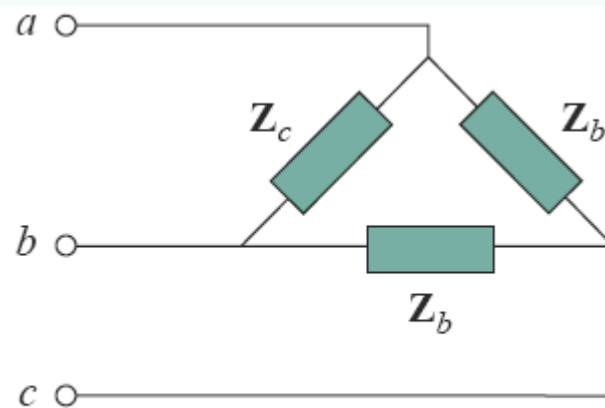


(b)



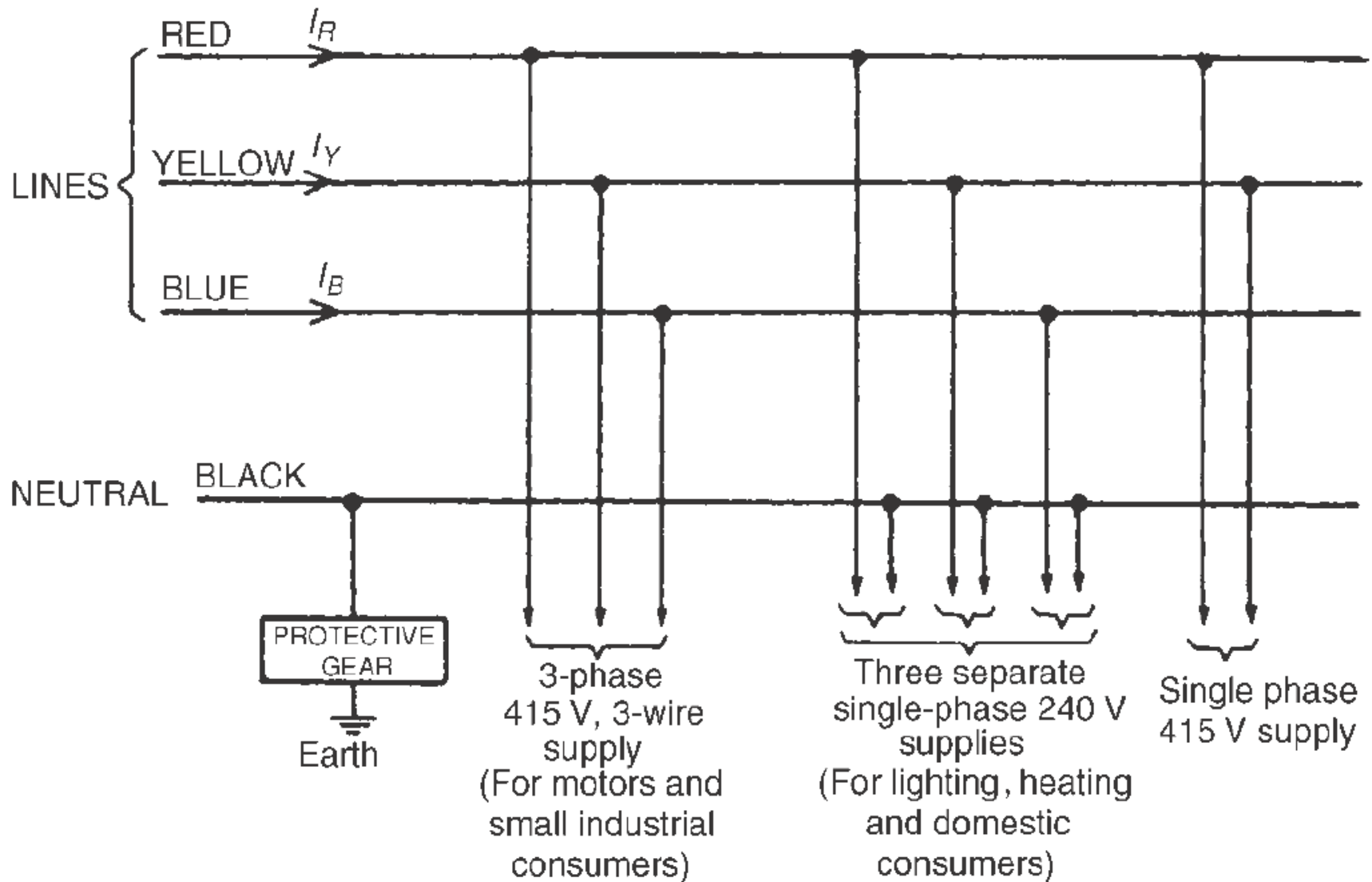
(a)

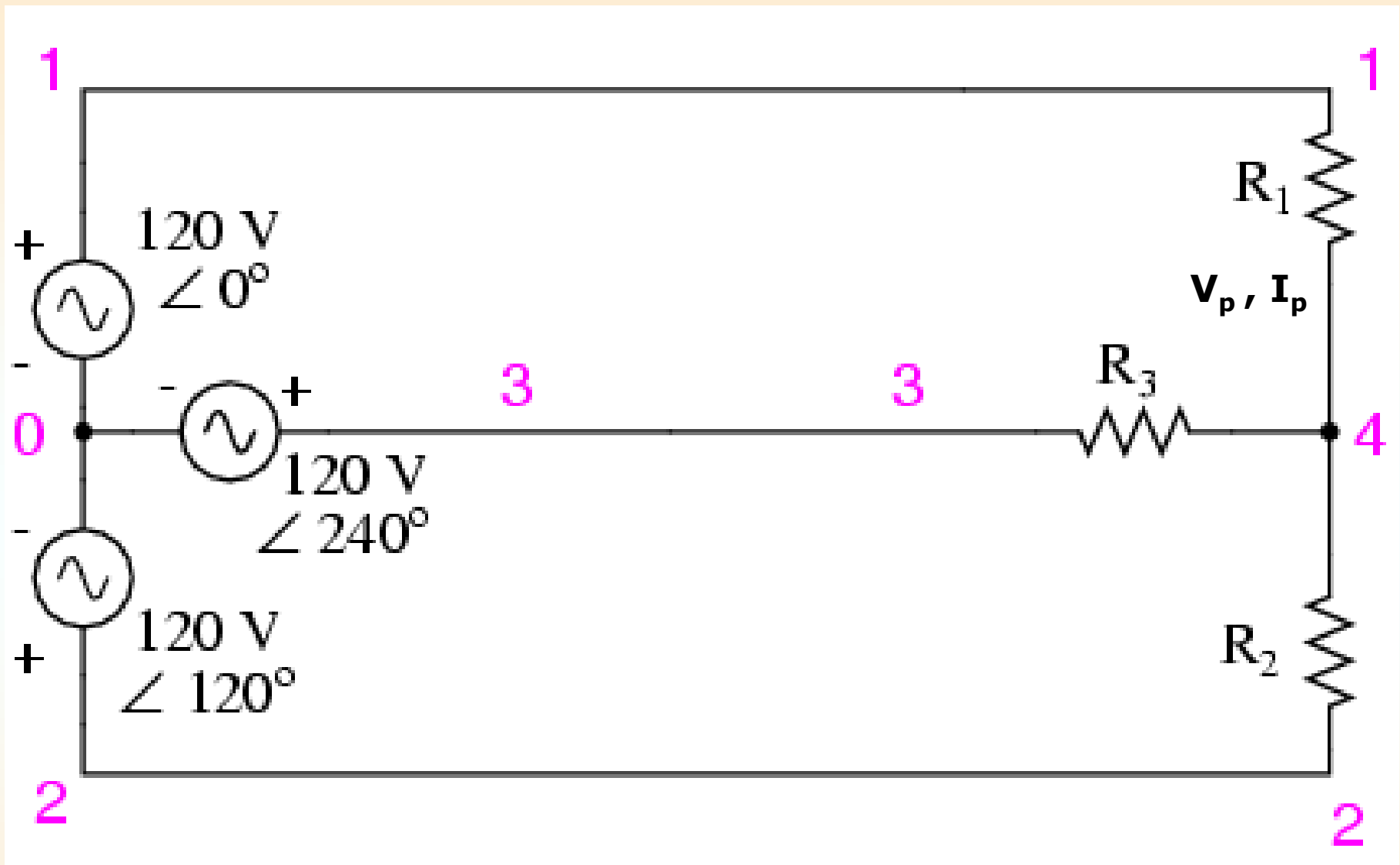
Three phase load



(b)

Three phase four wire 415 V, 50Hz supply





For balanced three phase load

$$P_1 = P_2 = P_3 = P$$

Total power consumed by three phase load = 3 X P

For unbalanced three phase load

$$P_1 \neq P_2 \neq P_3$$

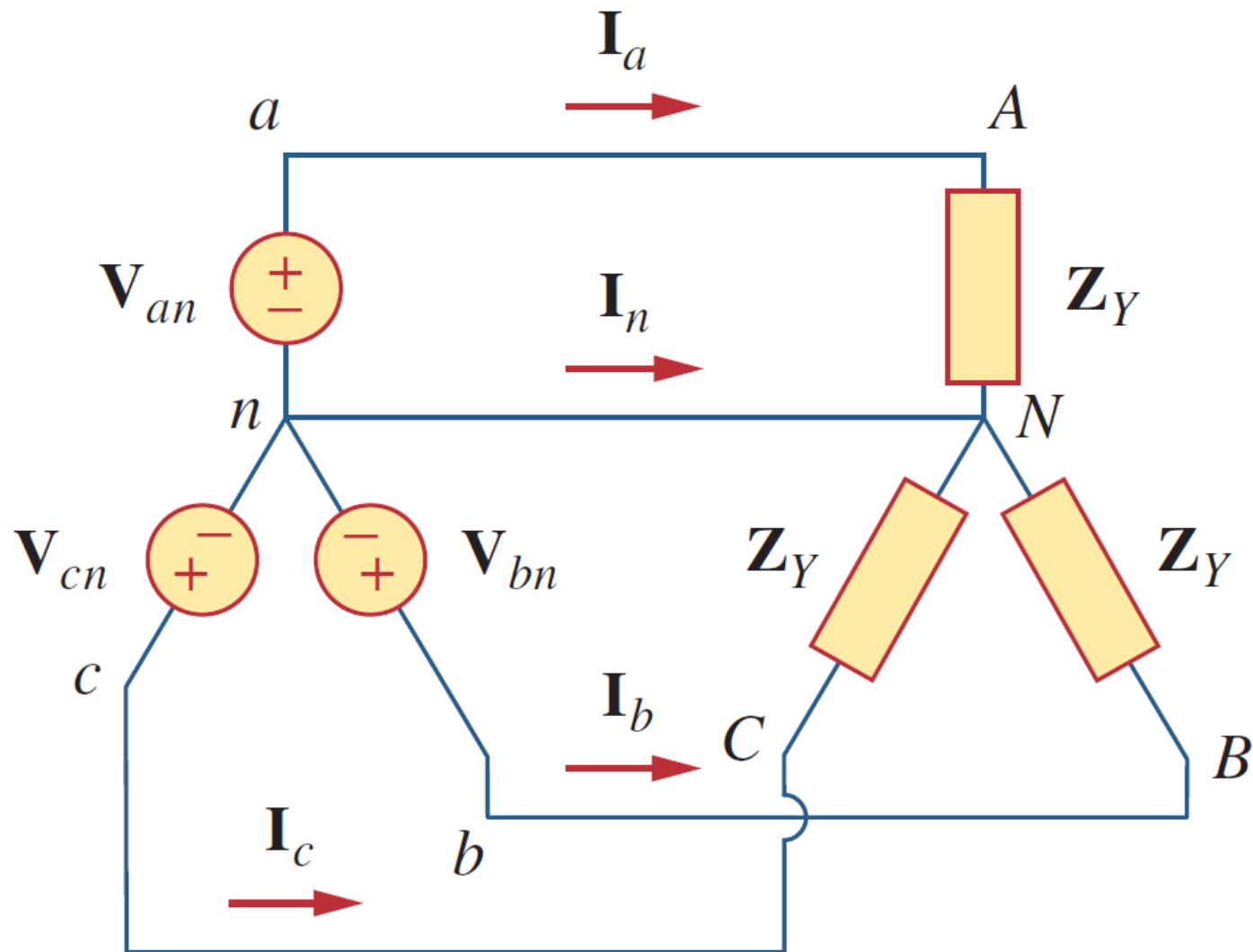
Total power consumed by three phase load = $P_1 + P_2 + P_3$

In terms of impedance, the complex power consumed by the load is

$$\mathbf{S} = I_{\text{rms}}^2 \mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*}$$

In terms of voltage and current

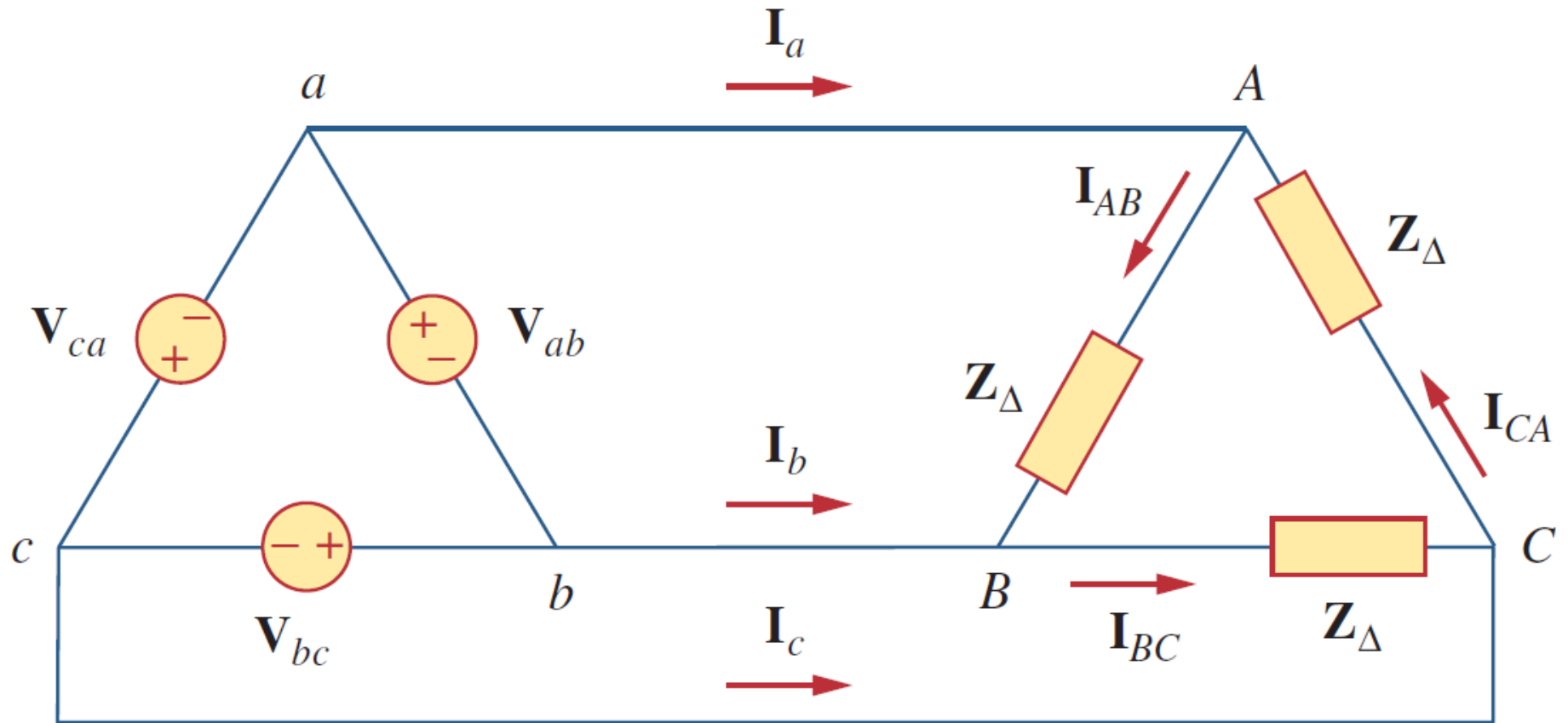
$$\begin{aligned} \frac{1}{2} \mathbf{V} \mathbf{I}^* &= \frac{1}{2} V_m I_m \angle \theta_v - \theta_i \\ &= \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)] \end{aligned}$$



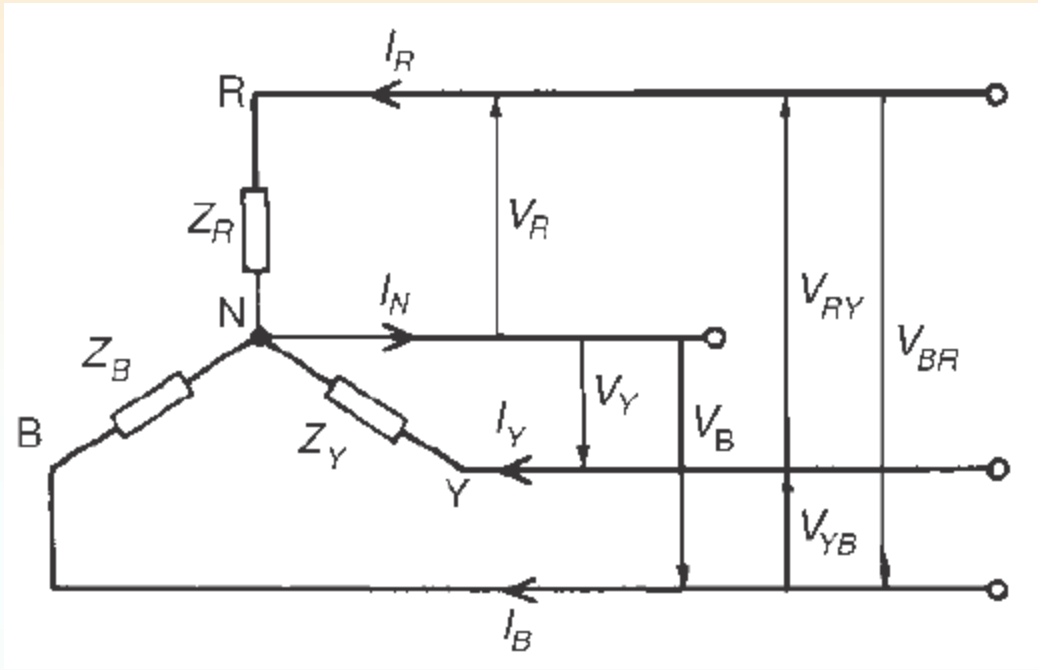
$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ, \quad V_{cn} = V_p \angle +120^\circ$$

$$\begin{aligned}\mathbf{I}_a &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle -240^\circ) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ\end{aligned}$$



Star connection



$$V_R = V_P \angle 0^\circ$$

$$V_Y = V_P \angle -120^\circ$$

$$V_B = V_P \angle -240^\circ$$

- (i) The three line conductors are connected to a load and the outlets from the loads are joined together at N to form the **neutral point** or the **star point**.
- (ii) The voltages, V_R , V_Y and V_B are called **phase voltages** or line to neutral voltages. Phase voltages are generally denoted by V_p .
- (iii) The voltages, V_{RY} , V_{YB} and V_{BR} are called **line voltages**.

(iv) The phase currents (generally denoted by I_p) are equal to their respective line currents I_R , I_Y and I_B , i.e. for a star connection:

$$I_L = I_p$$

(v) For a balanced system: $I_R = I_Y = I_B$, $V_R = V_Y = V_B$

$$V_{RY} = V_{YB} = V_{BR}, \quad Z_R = Z_Y = Z_B$$

and the current in the neutral conductor, $I_N = 0$

When a star connected system is balanced, then the neutral conductor is unnecessary and is often omitted.

(vi) The line voltage, V_{RY} ,

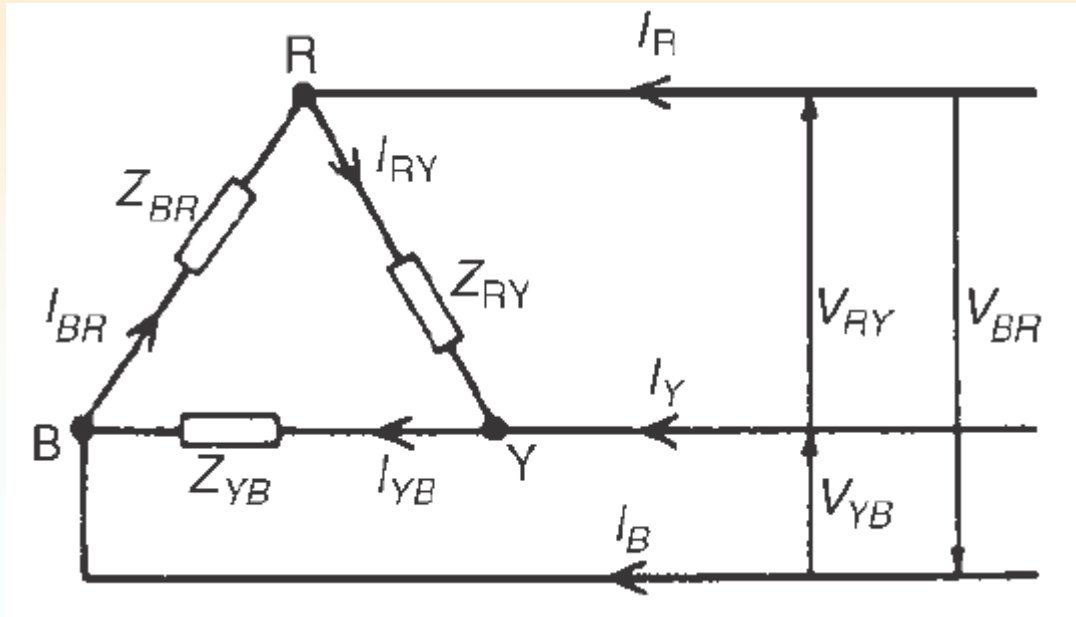
$\mathbf{V}_{RY} = \mathbf{V}_R - \mathbf{V}_Y$ (V_Y is negative since it is in the opposite direction to V_{RY}).

$$= V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$= V_p \left(1 + \frac{1}{2} + j \frac{\sqrt{3}}{2} \right) = \sqrt{3} V_p \angle 30^\circ$$

$$V_L = \sqrt{3} V_p$$

Delta connection



$$I_{RY} = I_P \angle 0^\circ$$

$$I_{YB} = I_P \angle -120^\circ$$

$$I_{BR} = I_P \angle -240^\circ$$

(i) It can be seen that the line voltages V_{RY} , V_{YB} and V_{BR} are the respective phase voltages, i.e. for a delta connection:

$$V_L = V_p$$

(ii) $I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$

$$= I_P \angle 0^\circ - I_P \angle -240^\circ = \sqrt{3} I_P \angle -30^\circ$$

$$I_L = \sqrt{3} I_p$$

POWER IN A BALANCED SYSTEM

For a Y-connected load, the phase voltages are

$$v_{AN} = \sqrt{2}V_p \cos \omega t,$$

$$v_{BN} = \sqrt{2}V_p \cos(\omega t - 120^\circ)$$

V_p has been defined as the rms value of the phase voltage.

If $\underline{Z}_Y = Z \angle \theta$, the phase currents lag behind their corresponding phase voltages by θ . Thus,

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta),$$

$$i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ)$$

The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$p = p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c$$

$$\begin{aligned} = 2V_p I_p [& \cos \omega t \cos(\omega t - \theta) \\ & + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) \\ & + \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ)] \end{aligned}$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$\begin{aligned} p &= V_p I_p [3 \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\ &\quad + \cos(2\omega t - \theta + 240^\circ)] \\ &= V_p I_p [3 \cos \theta + \cos \alpha + \cos \alpha \cos 240^\circ + \sin \alpha \sin 240^\circ \\ &\quad + \cos \alpha \cos 240^\circ - \sin \alpha \sin 240^\circ] \\ &\quad \text{where } \alpha = 2\omega t - \theta \\ &= V_p I_p \left[3 \cos \theta + \cos \alpha + 2 \left(-\frac{1}{2} \right) \cos \alpha \right] = 3 V_p I_p \cos \theta \end{aligned}$$

The average power per phase P_p for either the Δ -connected load or the Y-connected load is $p/3$, or

$$P_p = V_p I_p \cos \theta$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin \theta$$

The apparent power per phase is

$$S_p = V_p I_p$$

The complex power per phase is

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^*$$

The total average power is the sum of the average powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos \theta = \sqrt{3} V_L I_L \cos \theta$$

Similarly, the total reactive power is

$$Q = 3V_p I_p \sin \theta = 3Q_p = \sqrt{3}V_L I_L \sin \theta$$

and the total complex power is

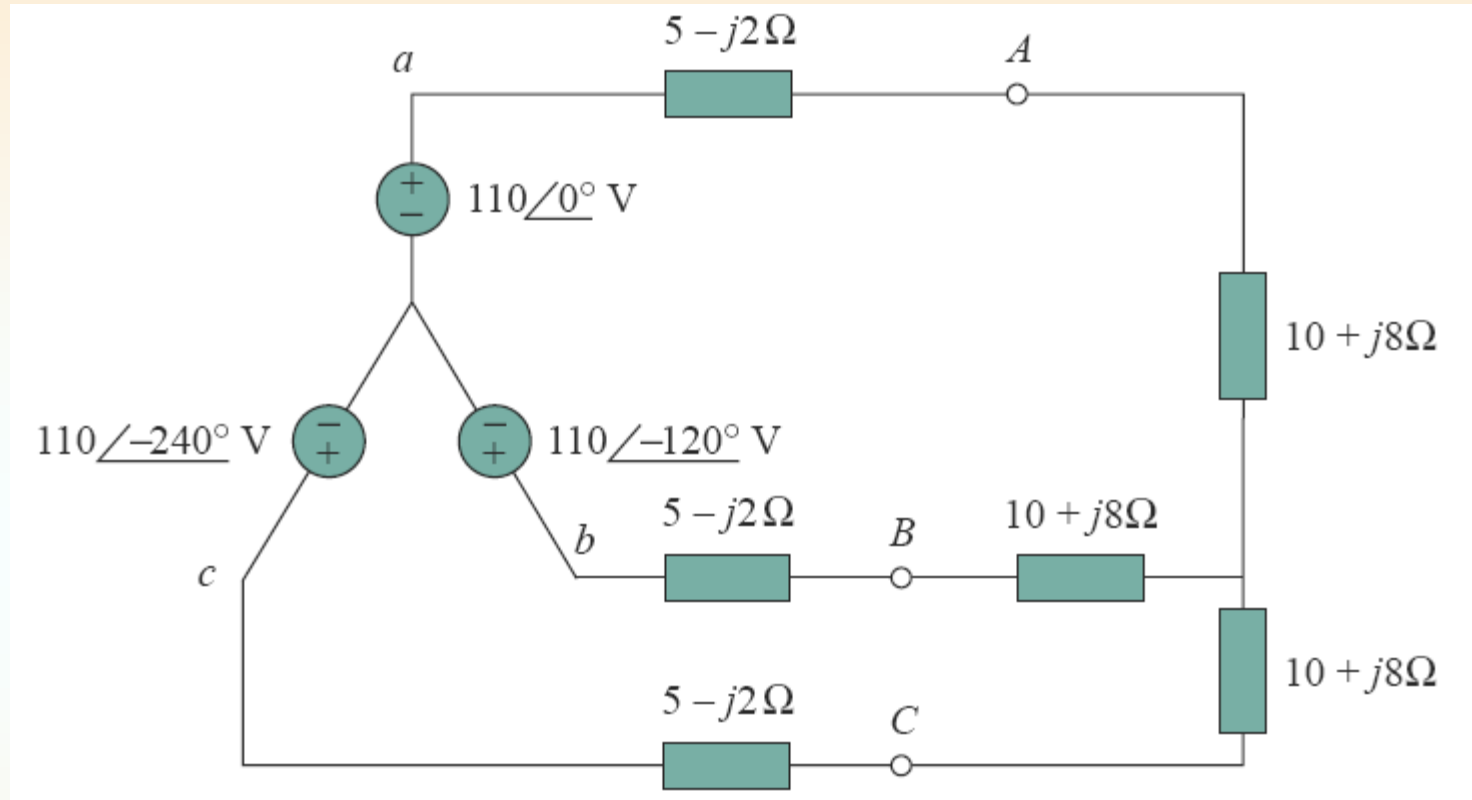
$$S = 3S_p = 3\mathbf{V}_p \mathbf{I}_p^* = 3I_p^2 \mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*}$$

or

$$S = P + jQ = \sqrt{3}V_L I_L \angle \theta$$

P36.

Calculate the line currents in the three-wire Y-Y system of Fig.



$$I_a = \frac{V_{an}}{Z_Y}$$

where

$$Z_Y = (5 - j2) + (10 + j8)$$

$$= 15 + j6 = 16.155\angle 21.8^\circ.$$

$$\mathbf{I}_a = \frac{110 \angle 0^\circ}{16.155 \angle 21.8^\circ} = 6.81 \angle -21.8^\circ \text{ A}$$

Since the source voltages in Fig. are in positive sequence and the line currents are also in positive sequence,

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = 6.81 \angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle -240^\circ = 6.81 \angle -261.8^\circ \text{ A} = 6.81 \angle 98.2^\circ \text{ A}$$

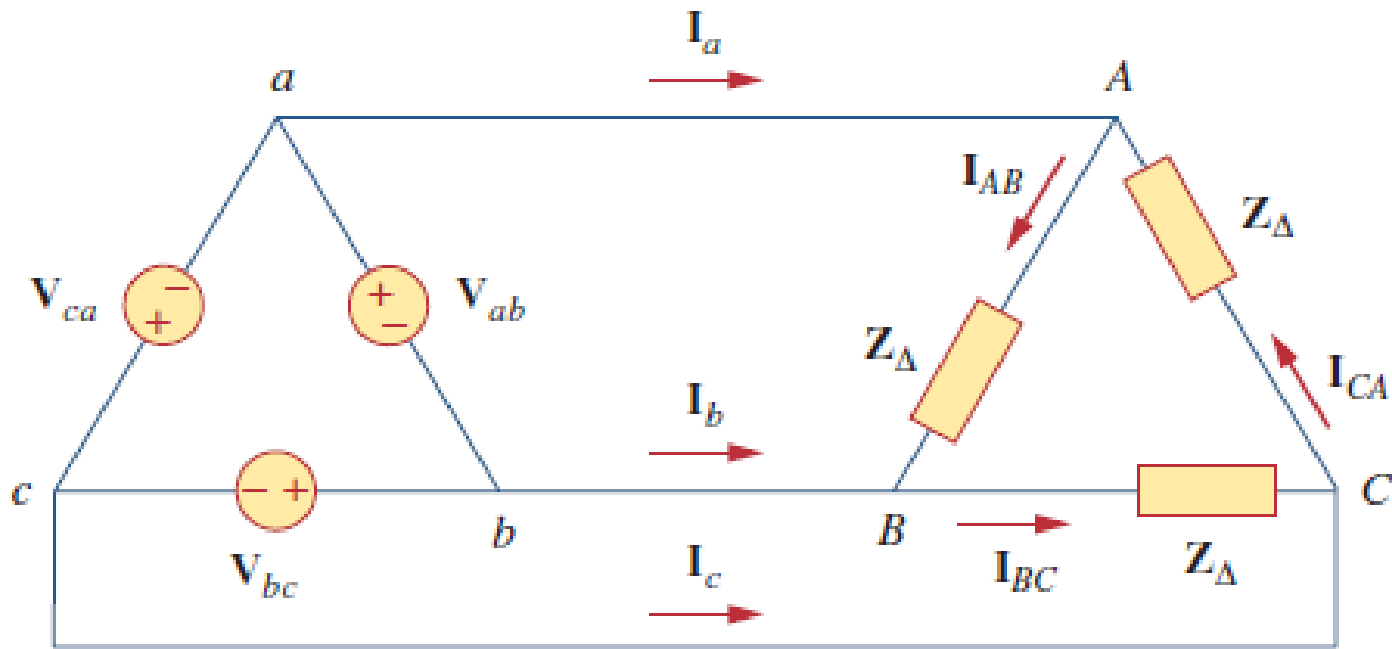


Figure 12.17
A balanced Δ - Δ connection.

Assuming a positive sequence, the phase voltages for a delta-connected source are

$$\begin{aligned} \mathbf{V}_{ab} &= V_p \angle 0^\circ \\ \mathbf{V}_{bc} &= V_p \angle -120^\circ, \quad \mathbf{V}_{ca} = V_p \angle +120^\circ \end{aligned} \quad (12.29)$$

The line voltages are the same as the phase voltages. From Fig. 12.17, assuming there is no line impedances, the phase voltages of the delta-connected source are equal to the voltages across the impedances; that is,

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \quad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \quad \mathbf{V}_{ca} = \mathbf{V}_{CA} \quad (12.30)$$

Hence, the phase currents are

$$\begin{aligned} \mathbf{I}_{AB} &= \frac{\mathbf{V}_{AB}}{Z_\Delta} = \frac{\mathbf{V}_{ab}}{Z_\Delta}, \quad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{Z_\Delta} = \frac{\mathbf{V}_{bc}}{Z_\Delta} \\ \mathbf{I}_{CA} &= \frac{\mathbf{V}_{CA}}{Z_\Delta} = \frac{\mathbf{V}_{ca}}{Z_\Delta} \end{aligned} \quad (12.31)$$

Since the load is delta-connected just as in the previous section, some of the formulas derived there apply here. The line currents are obtained from the phase currents by applying KCL at nodes A , B , and C , as we did in the previous section:

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \quad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \quad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \quad (12.32)$$

Also, as shown in the last section, each line current lags the corresponding phase current by 30° ; the magnitude I_L of the line current is $\sqrt{3}$ times the magnitude I_p of the phase current,

$$I_L = \sqrt{3} I_p \quad (12.33)$$

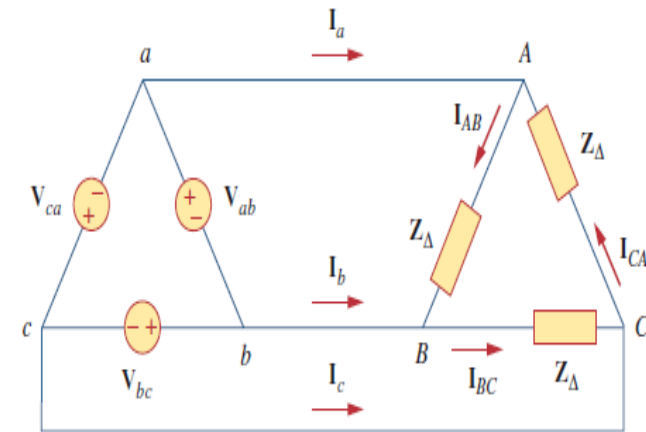


Figure 12.17
A balanced Δ - Δ connection.

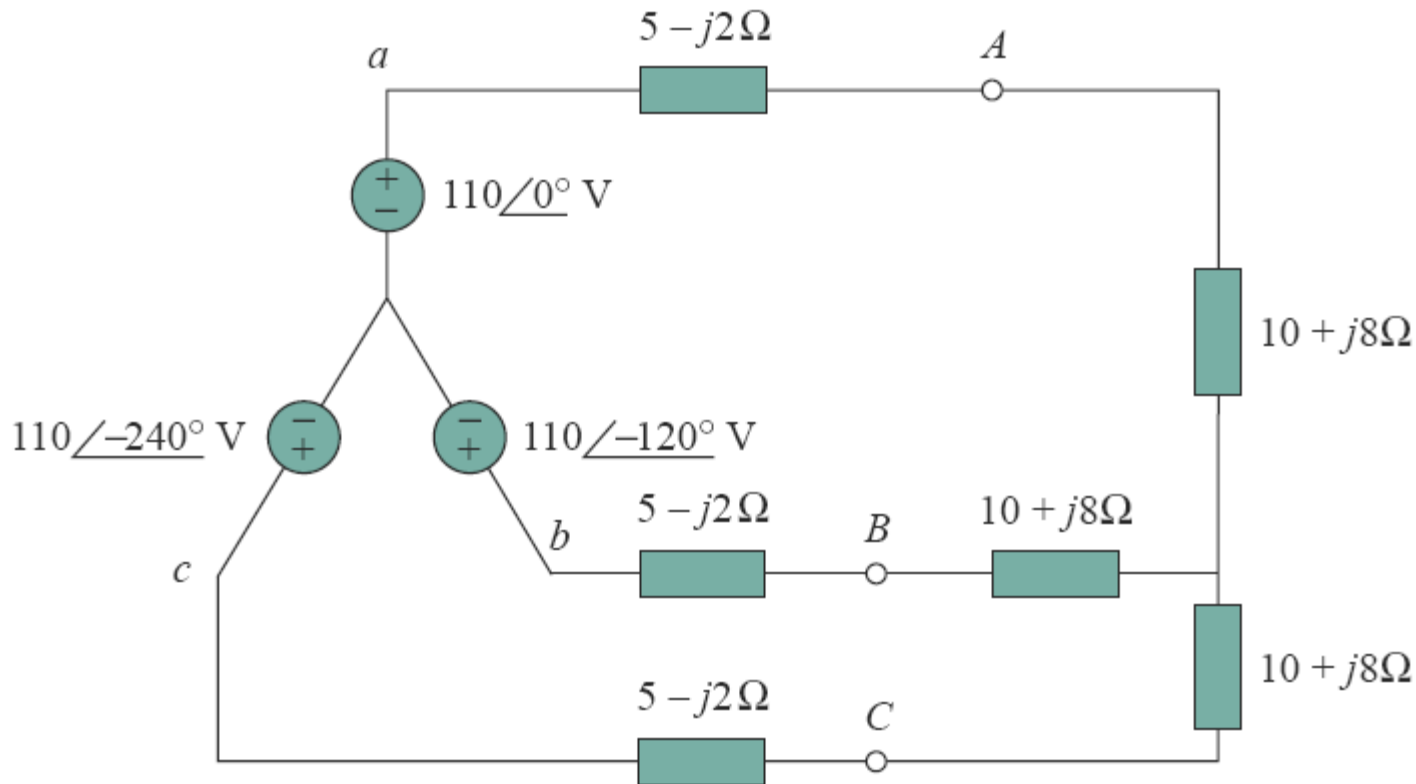
P38.

A positive-sequence, balanced Δ -connected source supplies a balanced Δ -connected load. If the impedance per phase of the load is $18 + j12 \Omega$ and $\mathbf{I}_a = 22.5 \angle 35^\circ \text{ A}$, find $\mathbf{I}_{AB}$ and $\mathbf{V}_{AB}$.

Answer: $13 \angle 65^\circ \text{ A}$, $281.2 \angle 98.69^\circ \text{ V}$.

P40.

Determine the total average power, reactive power, and complex power at the source and at the load.



$$\mathbf{V}_p = 110\angle 0^\circ \text{ V} \quad \text{and} \quad \mathbf{I}_p = 6.81\angle -21.8^\circ \text{ A}$$

At the load, the complex power absorbed is

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2 \mathbf{Z}_p$$

where $\mathbf{Z}_p = 10 + j8 = 12.81 \angle 38.66^\circ$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81 \angle -21.8^\circ$.
Hence

$$\begin{aligned}\mathbf{S}_L &= 3(6.81)^2 12.81 \angle 38.66^\circ = 1782 \angle 38.66^\circ \\ &= (1392 + j1113) \text{ VA}\end{aligned}$$

P44.

Three identical coils, each of resistance $15\ \Omega$ and inductance $42\ \text{mH}$ are connected (a) in star and (b) in delta to a $415\ \text{V}$, $50\ \text{Hz}$, 3-phase supply. Determine the total power dissipated in each case.

(a) **Star connection**

$$\begin{aligned}\text{Inductive reactance } X_L &= 2\pi fL = 2\pi(50)(42 \times 10^{-3}) \\ &= 13.19 \, \Omega\end{aligned}$$

$$\begin{aligned}\text{Phase impedance } Z_p &= \sqrt{(R^2 + X_L^2)} = \sqrt{(10^2 + 13.19^2)} \\ &= 16.55 \, \Omega\end{aligned}$$

Line voltage $V_L = 415 \, \text{V}$ and

$$\text{phase voltage, } V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240 \, \text{V}$$

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{240}{16.55} = 14.50 \, \text{A}$$

Line current, $I_L = I_p = 14.50 \, \text{A}$

$$\text{Power factor} = \cos \phi = \frac{R_p}{Z_p} = \frac{10}{16.55} = 0.6042 \text{ lagging}$$

$$\begin{aligned}\text{Power dissipated, } P &= \sqrt{3}V_L I_L \cos \phi = \sqrt{3}(415)(14.50)(0.6042) \\ &= \mathbf{6.3 \, kW}\end{aligned}$$

$$(\text{Alternatively, } P = 3I_p^2 R_p = 3(14.50)^2(10) = \mathbf{6.3 \, kW})$$

(b) **Delta connection**

$$V_L = V_p = 415 \text{ V}, Z_p = 16.55 \text{ } \Omega,$$
$$\cos \phi = 0.6042 \text{ lagging (from above).}$$

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{415}{16.55} = 25.08 \text{ A}$$

$$\text{Line current, } I_L = \sqrt{3}I_p = \sqrt{3}(25.08) = 43.44 \text{ A}$$

$$\text{Power dissipated, } P = \sqrt{3}V_L I_L \cos \phi = \sqrt{3}(415)(43.44)(0.6042)$$
$$= \mathbf{18.87 \text{ kW}}$$

$$(\text{Alternatively, } P = 3I_p^2 R_p = 3(25.08)^2(10) = \mathbf{18.87 \text{ kW}})$$

