REG.NO.:

VIT®
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - II
FALL SEMESTER 2026-2027

SLOT: E2+TE2

Programme Name & Branch : **B.Tech., Common to all branch**
Course Code and Course Name : **BAMAT101, Multivariable Calculus and Differential Equations**
Date of Examination : **01/10/2026**
Exam Duration : **90 minutes** **Maximum Marks: 50**

General instruction(s): Open Text Book / Open Note Book mode

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply)

Course Outcomes

2. Solve multiple integrals in cartesian, polar, cylindrical and spherical coordinates.
3. Understand gradient, directional derivatives, divergence, curl, Green's, Stokes and Gauss Divergence theorems.
4. Utilize different solution methods to obtain the solution for second-order ordinary differential equations.

Q. No	Question	M	CO	BL
1.	A solid tumor occupies a region V in the body modelled by the bounds $0 \le x \le 2$, $0 \le y \le \sqrt{4-x^2}$, and $0 \le z \le 4 - x^2 - y^2$. Because of poor vascularization at the core, an injected therapeutic agent distributes unevenly. The concentration density of the drug at any point is given by $f(x,y,z) = z$ mg/cm^3. Evaluate $\iiint_V f\, dv$ to find the total mass of the drug that successfully accumulated inside the tumor volume.	10	2	2
2.	A spherical biological cell has a dense conductive nucleus at the center, surrounded by an insulating lipid bilayer membrane shell. The membrane occupies the region bounded by an inner radius of a = 2μm and an outer radius of b = 4μm. Due to the ion pumps maintaining a membrane potential, a strong radial electric field is established across this shell. The resulting electrostatic energy density drops off heavily with distance from the center and is given by the function $f(x,y,z) = \frac{1}{(x^2+y^2+z^2)^2}$. Evaluate the triple integral over the entire spherical membrane volume V to find the total stored electrostatic energy in the cell's membrane.	10	2	2
3.	In neurophysics, the extracellular electrical environment around a firing pyramidal neuron is monitored using multi-electrode arrays. The electrostatic scalar potential *V (x, y, z)* (in millivolts, mV) in the surrounding brain tissue is mapped by the function: $V(x,y,z) = 4x^2 - 3xy + z^2$where *x, y,* and z represent spatial dimensions measured in micrometres (μm). Find the electric field vector function $E(x, y, z) = -\nabla \cdot V$ for this region. Calculate the directional derivative of the electric field's x-component (E_x) at the specific point P(1, 2, −1) in the direction pointing towards a adjacent node at Q(3, 3, 1).	10	3	2
4.	Verify Green's Theorem for $\oint_C (x^2 - 2xy)dx + (x^2y + 3)dy$ around the boundary C of the region $y^2 = 8x$ and x = 2.	10	3	3
5.	Find the solution of the differential equation $\frac{d^2y}{dx^2} + 4y(x) = e^{-x} - 3x^3$ using undetermined coefficients.	10	4	3

Answer key :
BMAT101 - E2 + TE2 slot .

Le'

① Let $x = r\cos\theta$, $y = r\sin\theta$

Given $0 \le x \le 2$, $0 \le y \le \sqrt{4-x^2}$, $0 \le z \le 4-x^2-y^2$

$\Rightarrow \quad 0 \le z \le 4-r^2$, $0 \le r \le 2$, $0 \le \theta \le \pi/2$.

Evaluate $\iiint_V z\,dV = \int_0^{\pi/2}\int_0^2\int_0^{4-r^2} z\,r\,dz\,dr\,d\theta = \int_0^{\pi/2}\int_0^2 \frac{(4-r^2)^2}{2}\,dr\,d\theta$

$= \int_0^{\pi/2} \frac{16}{3}\,d\theta = \frac{8\pi}{3}$.

② $x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$, $dv = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$2 \le \rho \le 4$, $0 \le \phi \le \pi$, $0 \le \theta \le 2\pi$

Total energy $= \iiint_V \frac{1}{(x^2+y^2+z^2)^2}\,dV$

$= \int_0^{2\pi}\int_0^{\pi}\int_2^4 \frac{1}{\rho^4}\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{4}\sin\phi\,d\phi\,d\theta = \pi$.

③ Let $V = 4x^2 - 3xy + z^2$

$\frac{\partial V}{\partial x} = 8x - 3y$, $\frac{\partial V}{\partial y} = -3x$, $\frac{\partial V}{\partial z} = 2z$

$E = -\nabla V = -(8x-3y)\vec{i} + 3x\vec{j} - 2z\vec{k}$

$E_x = 3y - 8x$. & $P = (1,2,-1)$ & $Q = (3,3,1)$

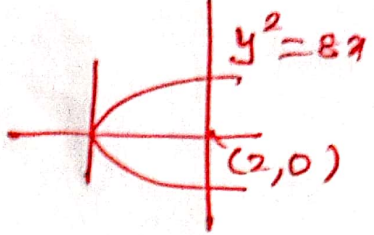
$\vec{PQ} = \vec{OQ} - \vec{OP} = 2\vec{i} + \vec{j} + 2\vec{k}$, $|\vec{PQ}| = 3$,

$u = \frac{\vec{PQ}}{|\vec{PQ}|} = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

$\nabla E_x = -8\vec{i} + 3\vec{j}$., $D_u(E_x) = \nabla E_x \cdot \vec{u} = \frac{-13}{3}$.

Scanned with OKEN Scanner

(4)



$P = x^2 - 2xy, \quad Q = x^2y + 3$

$$\iint_D \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] dA = \int_{-4}^{4} \int_{y^2/8}^{2} [2xy + 2x]\, dx\, dy$$

$$= \int_{-4}^{4} (y+1)\left[4 - \frac{y^4}{64}\right] dy = \frac{128}{5} = 25.6$$

$C = C_1 \cup C_2$: C_1 : along $x = 2$ from $-4 < y < 4$

C_2 : along $y^2 = 8x$. $\quad 4 \le y \le -4$

$$\int_{C_1} [P\,dx + Q\,dy] = \int_{C_1} Q\,dy = \int_{-4}^{4} [2y + 3]\, dy = 24$$

$$\int_{C_2} [P\,dx + Q\,dy] = \int_{4}^{-4} \left[\frac{y^3}{32} - \frac{y^4}{16} + \frac{y^3}{8} + 3\right] dy = \frac{8}{5}$$

$$\int_{C} [P\,dx + Q\,dy] = 24 + \frac{8}{5} = \frac{128}{5}$$

Scanned with OKEN Scanner

(5) Complementary function

$y'' + 4y = 0$

$m = \pm 2i, \quad y_c = c_1 \cos 2x + c_2 \sin 2x.$

Particular Integral: $y_{p_1} = Ae^{-x}$

$y'' + 4y = e^{-x} \Rightarrow 5Ae^{-x} = e^{-x}$

$\boxed{A = 1/5}$

$y_{p_2} = Ax^3 + Bx^2 + Cx + D.$

$y'' + 4y = -3x^3$

$4Ax^3 + 4Bx^2 + (6A + 4C)x + (2B + 4D) = -3x^3$

Coeff of x^3: $4A = -3$ $\quad \boxed{A = -3/4}$

Coeff of x^2: $4B = 0$ $\quad \boxed{B = 0}$

Coeff of x: $4C + 6A = 0$ $\quad \boxed{C = \frac{9}{8}}$

Constant term: $2B + 4D = 0 \Rightarrow \boxed{D = 0}$

$$y_{p_2} = \frac{-3}{4}x^3 + \frac{9}{8}x$$

$y = y_c + y_{p_1} + y_{p_2}.$

$$y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{5}e^{-x} - \frac{3}{4}x^3 + \frac{9}{8}x$$

is the General solution.

Scanned with OKEN Scanner