REG.NO.:

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SCHOOL OF ADVANCED SCIENCES
CONTINUOUS ASSESSMENT TEST - II
FALL SEMESTER 2026-2027

SLOT:E1+TE1

Programme Name & Branch : **B. Tech**
Course Code and Course Name : **BAMAT101L & Multivariable Calculus and Differential Equations**
Faculty Name(s) : **Common to all**
Class Number(s) : **Common to all**
Date of Examination : **01.10.2026 & 2.00 pm to 3.30 pm**
Exam Duration : **90 minutes** **Maximum Marks: 50**

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- CO2–Solve multiple integrals in cartesian, polar, cylindrical and spherical coordinates.
- CO3– Understand gradient, directional derivatives, divergence, curl, Green's, Stokes and Gauss Divergence theorems.
- CO4– utilize different solution methods to obtain the solution for second-order ordinary differential equations.

Q. No	Question	M	CO	BL
1.	Evaluate the integral $\iint_R (3xy + 2y)dA$, where $R = \{1 \leq x^2 + y^2 \leq 4, x \leq 0\}$ using polar coordinates.	10	2	3
2.	A computer vision system detects a cylindrical pipe of radius 3 units and height 4 units. Determine its total intensity if $f(x, y, z) = 2\,x^2 + y^2 + z$ by using cylindrical coordinates.	10	2	3
3.	An electromagnetic solver design framework models a static electric field vector inside a micro-sensor assembly via the function: $E = 2\cos y\ \hat{\imath} + \left(\frac{1}{y} - 2x\sin y\right)\ \hat{\jmath} + \frac{1}{z}\ \hat{k}$ Prove that this electric field is conservative. Extract the corresponding scalar electric potential function.	10	3	3
4.	Verify Green's theorem for the vector field $\vec{F} = (3y + \sin x)\hat{\imath} + (e^y - 2x)\hat{\jmath}$ over the region bounded by the circle $x^2 + y^2 = 4$.	10	3	3
5.	A stable second-order signal-processing system is described by the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = cos(3t)$. Using the method of undetermined coefficients, determine the complete response of the system and identify the transient response and steady-state response.	10	4	3

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CAT-2-Key

Tuesday, 15 September 2026 3:15 PM

1. Evaluate the integral $\iint_R (3xy + 2y)dA$, where $R = \{1 \le x^2 + y^2 \le 4, x \le 0\}$ using polar coordinates.

$x = r\cos\theta \qquad y = r\sin\theta \qquad dx\,dy = r\,dr\,d\theta$

$$\iint_R (3xy+2y)\,dA = \int_{\pi/2}^{3\pi/2}\int_1^2 (3r^2\cos\theta\sin\theta + 2r\sin\theta)\,r\,dr\,d\theta \quad \text{— 5M}$$

$$= \int_{\pi/2}^{3\pi/2} 3\frac{\sin 2\theta}{2}\left(\frac{r^4}{4}\right)_1^2 d\theta + \int_{\pi/2}^{3\pi/2} 2\sin\theta\left(\frac{r^3}{3}\right)_1^2 d\theta$$

$$= \frac{45}{8}\int_{\pi/2}^{3\pi/2} \sin 2\theta\,d\theta + \frac{14}{3}\int_{\pi/2}^{3\pi/2} \sin\theta\,d\theta$$

$$= \frac{45}{8}\left[-\frac{\cos 2\theta}{2}\right]_{\pi/2}^{3\pi/2} + \frac{14}{3}\left[-\cos\theta\right]_{\pi/2}^{3\pi/2}$$

$$= \frac{45}{8}\left[\frac{1}{2} - \frac{1}{2}\right] + \frac{14}{3}[0-0]$$

$$= 0 \quad \text{— 5M}$$

2. A computer vision system detects a cylindrical pipe of radius 3 units and height 4 units. Determine its total intensity if $f(x, y, z) = 2x^2 + y^2 + z$ by using cylindrical coordinates.

$$\text{Total intensity} = \iiint_V (2x^2+y^2+z)\,dz\,dy\,dx \quad \text{— 1M}$$

$$= \int_{\theta:0}^{2\pi}\int_{r:0}^{3}\int_{z:0}^{4} (2r^2\cos^2\theta + r^2\sin^2\theta + z)\,r\,dz\,dr\,d\theta \quad \text{— 4M}$$

$$= \int_{\theta:0}^{2\pi}\int_{r:0}^{3}\left[rz + \frac{rz^2}{2} + r^3\cos^2\theta\, z\right]_0^4 dr\,d\theta$$

$$= \int_{\theta:0}^{2\pi}\int_{r:0}^{3} (4r^3 + 8r + 4r^3\cos^2\theta)\,dr\,d\theta$$

$$= \int_{\theta:0}^{2\pi} \left[r^4 + 4r^2 + r^4 \cos^2\theta \right]_0^3 d\theta$$

$$= \int_{\theta:0}^{2\pi} (117 + 81\cos^2\theta)\, d\theta$$

$$= 234\pi + \frac{81}{2}\left(\theta + \frac{\sin 2\theta}{2}\right)_0^{2\pi}$$

$$= 234\pi + 81\pi$$

$$= 315\pi$$ —— 5M

3. An electromagnetic solver design framework models a static electric field vector inside a micro-sensor assembly via the function: $E = 2\cos y\ \hat{\imath} + \left(\frac{1}{y} - 2x\sin y\right)\hat{\jmath} + \frac{1}{z}\hat{k}$ Prove that this electric field is conservative. Extract the corresponding scalar electric potential function.

$$\nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2\cos y & \frac{1}{y} - 2x\sin y & \frac{1}{z} \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(-2\sin y - (-2\sin y))$$

$$= 0$$ —— 3M

$\therefore \bar{F}$ is conservative.

Let $\bar{F} = \nabla\phi$

$$\frac{\partial\phi}{\partial x} = 2\cos y \quad \text{—①} \qquad \frac{\partial\phi}{\partial y} = \frac{1}{y} - 2x\sin y \quad \text{—②} \qquad \frac{\partial\phi}{\partial z} = \frac{1}{z} \quad \text{—③}$$ —2M

Integrating ① w.r.t 'x'

$$\phi = 2x\cos y + f(y,z) \quad \text{—④}$$

diff ④ partially w.r.t 'y'

$$\frac{\partial\phi}{\partial y} = -2x\sin y + \frac{\partial f}{\partial y} \quad \text{—⑤}$$

equating ⑤ & ②

$$-2x\sin y + \frac{\partial f}{\partial y} = \frac{1}{y} - 2x\sin y$$

$\frac{\partial f}{\partial y} = \frac{1}{y}$ on $\int$ing $f(y,z) = \ln y + h(z)$

④ $\Rightarrow \phi = 2x\cos y + \ln y + h(z)$ —⑥

diff ⑥ partially w.r.t 'z'

$\frac{\partial \phi}{\partial z} = \frac{dh}{dz}$ —⑦

equating ⑦ & ③

$\frac{dh}{dz} = \frac{1}{z}$ on $\int$ing $h(z) = \ln z + c$

⑥ $\Rightarrow \phi = 2x\cos y + \ln y + \ln z + c$

4. Verify Green's theorem for the vector field $\vec{F} = (3y + \sin x)\hat{\imath} + (e^y - 2x)\hat{\jmath}$ over the region bounded by the circle $x^2 + y^2 = 4$.

Green's theorem: $\oint_C M\,dx + N\,dy = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy\,dx$

Given $\bar{F} = (3y + \sin x)\hat{i} + (e^y - 2x)\hat{j}$

$L.H.S = \oint_C (3y + \sin x)\,dx + (e^y - 2x)\,dy$

parametric form $x = 2\cos\theta \quad y = 2\sin\theta$

$= \int_0^{2\pi} (6\sin\theta + \sin(2\cos\theta))(-2\sin\theta\,d\theta) + (e^{2\sin\theta} - 4\cos\theta)2\cos\theta\,d\theta$

$= \int_0^{2\pi} (-12\sin^2\theta - 2\sin\theta\sin(2\cos\theta) + 2\cos\theta e^{2\sin\theta} - 8\cos^2\theta)\,d\theta$

$2\cos\theta = t \quad -2\sin\theta\,d\theta = dt \quad t = 2\sin\theta$

$= -\frac{12}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} + \int_2^2 \sin t\,dt + \int_0^0 e^t\,dt - \frac{8}{2}\left(\theta + \frac{\sin 2\theta}{2}\right)_0^{2\pi}$

$= -6 * 2\pi + 0 + 0 - 4 * 2\pi = -20\pi$

$M = 3y + \sin x, \frac{\partial M}{\partial y} = 3; \quad N = e^y - 2x, \frac{\partial N}{\partial x} = -2$

$R.H.S = -\iint 5\,dA$

$$= -5\int_{\theta:0}^{2\pi}\int_{r:0}^{2} r\,dr\,d\theta$$

$$= -5\int_0^{2\pi}\left(\frac{r^2}{2}\right)_0^2 d\theta$$

$$= -10\,[\theta]_0^{2\pi} = -20\pi$$

$\therefore$ Green's thm verified. — 5M

5. A stable second-order signal-processing system is described by the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = cos(3t)$. Using the method of undetermined coefficients, determine the complete response of the system and identify the transient response and steady-state response.

$$y'' + 3y' + 2y = \cos(3t)$$

$$y_c = C_1 e^{-2t} + C_2 e^{-t} \quad \text{— 3M}$$

$$y_p = A\cos 3t + B\sin 3t$$

$$y_p' = -3A\sin 3t + 3B\cos 3t$$

$$y_p'' = -9A\cos 3t - 9B\sin 3t \quad \text{— 2M}$$

$$-9A\cos 3t - 9B\sin 3t + 3(-3A\sin 3t + 3B\cos 3t) + 2(A\cos 3t + B\sin 3t) = \cos(3t)$$

'$\cos 3t$' $\quad -9A + 9B + 2A = 1$

'$\sin 3t$' $\quad -9B - 9A + 2B = 0 \Rightarrow -7B - 9A = 0$

$$\Rightarrow A = -\frac{7}{9}B$$

$$-7A + 9B = 1$$

$$130B = 9 \Rightarrow B = \frac{9}{130} \quad A = -\frac{7}{130}$$

$$y_p = -\frac{7}{130}\cos 3t + \frac{9}{130}\sin 3t \quad \text{— 4M}$$

$$y = y_c + y_p = C_1 e^{-2t} + C_2 e^{-t} - \frac{7}{130}\cos 3t + \frac{9}{130}\sin 3t$$

— 1M