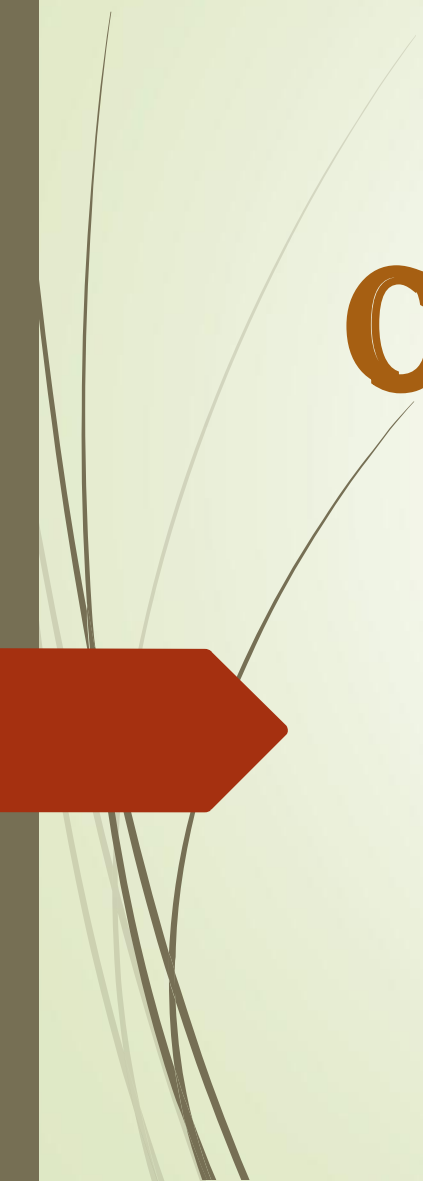




BEEE203L

CIRCUIT THEORY

DR. P. VIJAYAPRIYA
PROFESSOR/ SELECT

MODULE – IV

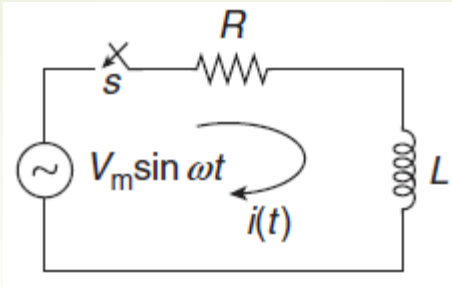
Analysis of Transient Response of Circuits

Contents

- Review of Laplace transformation;
- Laplace transform of network
- Time domain solution for RL,RC, RLC networks for DC excitation
- Time domain solution for RL,RC, RLC networks for AC excitation
- Transient Behaviour of circuit elements under switching conditions and their representations
- Evaluation of initial and final conditions in RL,RC,RLC circuits with DC excitation
- Evaluation of initial and final conditions in RL,RC,RLC circuits with AC excitation

Sinusoidal response of RL circuit

4



For the RL circuit shown

Applying KVL, the following equation is obtained:

$$V_m \sin \omega t = Ri(t) + L \frac{di}{dt}$$

Taking the Laplace transform on both sides, we get the equation as follows:

$$V_m L [\sin \omega t] = RI(s) + LsI(s)$$

$$V_m \frac{\omega}{s^2 + \omega^2} = (R + sL)I(s)$$

$$I(s) = \frac{\omega V_m}{(s^2 + \omega^2)(R + sL)}$$

$$= \frac{\omega V_m}{(s + j\omega)(s - j\omega)(R + sL)}$$

$$I(s) = \frac{\omega V_m}{L} \frac{1}{(s + j\omega)(s - j\omega) \left(s + \frac{R}{L} \right)}$$

Now, using the partial fractions, we get the equation as follows:

$$\frac{1}{(s + j\omega)(s - j\omega) \left(s + \frac{R}{L} \right)} = \frac{A}{(s + j\omega)} + \frac{B}{(s - j\omega)} + \frac{C}{\left(s + \frac{R}{L} \right)}$$

Substituting the above constants

$$A = \frac{L}{-j2\omega(R - j\omega L)}$$

$$B = \frac{L}{j2\omega(R + j\omega L)}$$

$$C = \frac{L^2}{R^2 + \omega^2 L^2}$$

So

$$I(s) = \frac{\omega V_m}{L} \left[\frac{\frac{L}{-j2\omega(R - j\omega L)}}{s + j\omega} + \frac{\frac{L}{j2\omega(R + j\omega L)}}{s - j\omega} + \frac{\frac{L^2}{R^2 + \omega^2 L^2}}{s + \frac{R}{L}} \right]$$

$$= V_m \left[\frac{\frac{1}{-j2(R - j\omega L)}}{s + j\omega} + \frac{\frac{1}{j2(R + j\omega L)}}{s - j\omega} + \frac{\frac{\omega L}{R^2 + \omega^2 L^2}}{s + \frac{R}{L}} \right]$$

$$I(s) = \frac{V_m}{-j2(s + j\omega)(R - j\omega L)} + \frac{V_m}{j2(s - j\omega)(R + j\omega L)} + \frac{V_m \omega L}{(R^2 + \omega^2 L^2) \left(s + \frac{R}{L} \right)}$$

Taking inverse Laplace Transform,

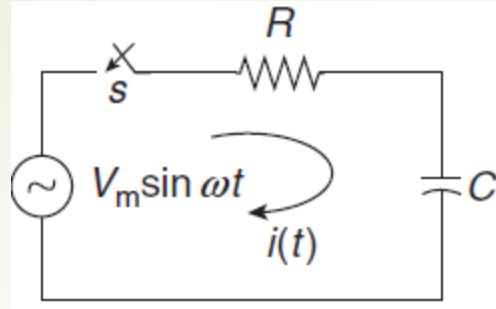
$$\begin{aligned} i(t) &= \frac{V_m}{-j2(R - j\omega L)} \cdot e^{-j\omega t} + \frac{V_m}{j2(R + j\omega L)} \cdot e^{j\omega t} + \frac{V_m \omega L}{(R^2 + \omega^2 L^2)} \cdot e^{-\frac{R}{L}t} \\ &= \frac{V_m}{-j2} \cdot \frac{(R + j\omega L)}{(R - j\omega L)(R + j\omega L)} e^{-j\omega t} + \frac{V_m (R - j\omega L)}{j2(R + j\omega L)(R - j\omega L)} e^{j\omega t} + \frac{V_m \omega L}{(R^2 + \omega^2 L^2)} e^{-\frac{R}{L}t} \\ &= j \frac{V_m (R + j\omega L)}{2(R^2 + \omega^2 L^2)} e^{-j\omega t} - j \frac{V_m (R - j\omega L)}{2(R^2 + \omega^2 L^2)} e^{j\omega t} + \frac{V_m \omega L}{(R^2 + \omega^2 L^2)} e^{-\frac{R}{L}t} \\ &= \frac{V_m}{2\sqrt{R^2 + \omega^2 L^2}} \left[\frac{j(R + j\omega L)}{\sqrt{R^2 + \omega^2 L^2}} e^{-j\omega t} - j \frac{(R - j\omega L)}{\sqrt{R^2 + \omega^2 L^2}} e^{j\omega t} \right] + \frac{V_m (\omega L)}{(R^2 + \omega^2 L^2)} e^{-\frac{R}{L}t} \end{aligned}$$

by using Euler's identity $e^{j\theta} = \cos(\theta) + j \sin(\theta)$ the equation becomes

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin \left(\omega t - \tan^{-1} \frac{\omega L}{R} \right) + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin \left(\tan^{-1} \frac{\omega L}{R} \right) e^{-\frac{R}{L}t}$$

Sinusoidal response of RC circuit

7



For the RC circuit shown

Applying KVL, the following equation is obtained:

$$V_m \sin \omega t = Ri(t) + \frac{1}{C} \int i dt$$

Taking the Laplace transform on both sides, we get the equation as follows:

$$\begin{aligned} V_m \cdot \frac{\omega}{s^2 + \omega^2} &= RI(s) + \frac{1}{C} \cdot \frac{I(s)}{s} \\ &= \left[R + \frac{1}{Cs} \right] I(s) \end{aligned}$$

$$I(s) = \frac{V_m \cdot \omega Cs}{(s^2 + \omega^2)(RCs + 1)}$$

$$I(s) = \frac{V_m \cdot \omega C s}{RC(s^2 + \omega^2) \left(s + \frac{1}{RC} \right)}$$

$$= \frac{V_m \omega}{R} \cdot \frac{s}{(s + j\omega)(s - j\omega) \left(s + \frac{1}{RC} \right)}$$

Now, using the partial fractions, we get the equation as follows:

$$I(s) = \frac{V_m \omega}{R} \cdot \left[\frac{A}{s + j\omega} + \frac{B}{s - j\omega} + \frac{C}{s + \frac{1}{RC}} \right]$$

Substituting the above constants

$$A = \frac{RC}{2(1 - j\omega RC)}$$

$$B = \frac{RC}{2(1 + j\omega RC)}$$

$$C = \frac{-RC}{1 + \omega^2 R^2 C^2}$$

So

$$I(s) = \frac{V_m \cdot \omega}{R} \cdot \left[\frac{RC}{2(1 - j\omega RC)} \cdot \frac{1}{s + j\omega} + \frac{RC}{2(1 + j\omega RC)} \cdot \frac{1}{s - j\omega} + \frac{-RC}{1 + \omega^2 R^2 C^2} \cdot \frac{1}{s + \frac{1}{RC}} \right]$$

$$I(s) = \frac{V_m \omega C}{2(1-j\omega RC)} \cdot \frac{1}{s+j\omega} + \frac{V_m \omega C}{2(1+j\omega RC)} \cdot \frac{1}{s-j\omega} - \frac{V_m \omega C}{1+\omega^2 R^2 C^2} \cdot \frac{1}{s+\frac{1}{RC}}$$

Taking inverse Laplace Transform,

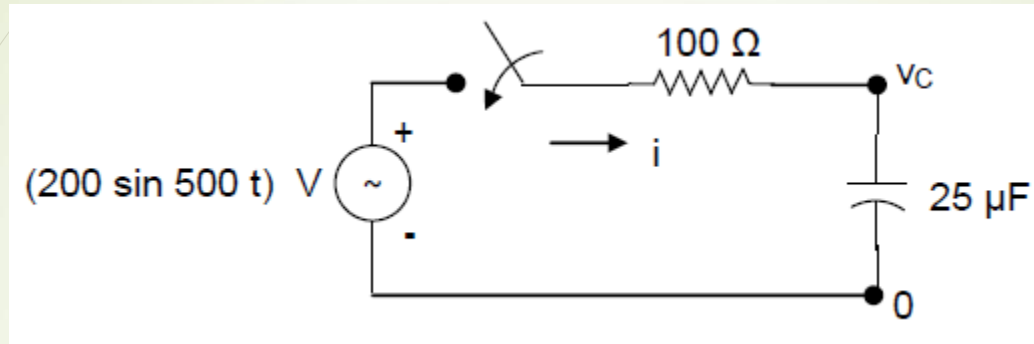
$$i(t) = \frac{V_m \omega C}{2(1-j\omega RC)} e^{-j\omega t} + \frac{V_m \omega C}{2(1+j\omega RC)} e^{j\omega t} - \frac{V_m \omega C}{1+\omega^2 R^2 C^2} e^{-\frac{t}{RC}}$$

Simplifying further

$$i(t) = \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \times \left[\sin\left(\omega t + \tan^{-1} \frac{1}{\omega CR}\right) - \sin\left(\tan^{-1} \frac{1}{\omega CR}\right) e^{-\frac{t}{RC}} \right]$$

P.4.21**10**

For the circuit shown below, find the transient current, assuming that the initial charge on the capacitor is zero, when the switch is closed at time $t = 0$

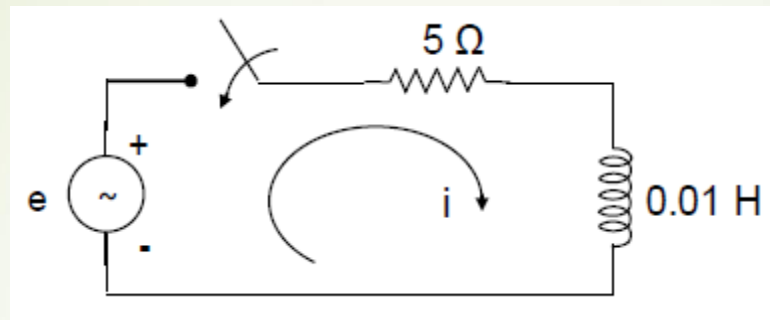


$$i(t) = \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \times \left[\sin\left(\omega t + \tan^{-1} \frac{1}{\omega CR}\right) - \sin\left(\tan^{-1} \frac{1}{\omega CR}\right) e^{-\frac{t}{RC}} \right]$$

$$i(t) = 1.5617 \sin(500 t + 38.66^\circ) - 0.9756 e^{-400t} \text{ A}$$

P.4.22 In the initially relaxed RL circuit shown, the sinusoidal source of $e = 100 \sin (500 t)$ V is applied at time $t = 0$. Determine the resulting transient current for time $t > 0$.

11



$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t - \tan^{-1} \frac{\omega L}{R}\right) + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\tan^{-1} \frac{\omega L}{R}\right) e^{-\frac{R}{L}t}$$

$$i(t) = 14.14 \sin (500 t - 45^\circ) + 10 e^{-500t} \text{ A}$$